Quiz 0 **Solutions**, Math 111, Section 5 (Vinroot)

Show all steps in each of the following.

(a): Determine if the following limit exists. If it exists, find the value, and if it does not but is infinite, then describe the infinite limit. Explain your answer.

$$\lim_{x \to \pi^+} \frac{2x}{\sin(x)}$$

Solution: First note that $\sin(\pi) = 0$, so when x approaches π from either direction, it appears the denominator approaches 0. The numerator approaches 2π as x approaches π from either direction. When $x > \pi$, but x is close to π, then $\sin(x) < 0$, which can be seen from either the unit circle or the graph of $y = \sin(x)$. This means that as $x \to \pi^+$, $\sin(x)$ approaches 0 but remains negative. Since the numerator remains positive, then the whole expression is negative. The reciprocal of a negative number close to 0 is a very negative number (that is, a negative number with large absolute value), then this means this expression approaches $-\infty$ as $x \to \pi^+$. Thus, the limit does not exist, but is infinite and

$$\lim_{x \to \pi^+} \frac{2x}{\sin(x)} = -\infty.$$

(b): If the function $g(x)$ is defined as follows, compute $\lim_{x \to 2} g(x)$ if it exists, and explain:

$$g(x) = \begin{cases} x^2 - 2 & \text{if } x \leq 2 \\ \frac{2}{x-1} & \text{if } x > 2. \end{cases}$$

Solution: The limit exists if and only if both $\lim_{x \to 2^-} g(x)$ and $\lim_{x \to 2^+} g(x)$ exist and are equal. We compute each of the one-sided limits as follows. When $x \to 2^-$, then $x < 2$, and $g(x) = x^2 - 2$. So we have, using this and the limit laws,

$$\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} (x^2 - 2) = \lim_{x \to 2^-} (x^2) - \lim_{x \to 2^-} 2 = \left(\lim_{x \to 2^-} x \right)^2 - 2 = 2^2 - 2 = 2.$$

When $x \to 2^+$, then $x > 2$, and $g(x) = 2/(x-1)$. Using this and limit laws, we compute

$$\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} \left(\frac{2}{x-1} \right) = \frac{\lim_{x \to 2^+} 2}{\lim_{x \to 2^+} (x-1)} = \frac{2}{\lim_{x \to 2^+} x - \lim_{x \to 2^+} 1} = \frac{2}{2-1} = 2.$$

Note that we have validly used the limit laws since the limits we computed exist and the denominator in the second limit was nonzero.

Since $\lim_{x \to 2^-} g(x) = 2 = \lim_{x \to 2^+} g(x)$, then the limit exists and $\lim_{x \to 2} g(x) = 2$.