Quiz 8 **Solutions**, Math 111, Section 2 (Vinroot)

Express the area under the curve $y = \sqrt{x}$ between $x = 2$ and $x = 5$ as the limit of a sum of rectangle areas using the right endpoint method. Draw a graph with an example of one of these rectangles.

Solution: We have $f(x) = \sqrt{x}$, and we want the area under $y = f(x)$ over the interval $[a, b]$ where $a = 2$ and $b = 5$. In the general notation, if we subdivide the interval $[a, b]$ into n equal subintervals, they each have length

$$\Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n}.$$

The right endpoint of the ith subinterval is then given by

$$x_i = a + i\Delta x = 2 + i\Delta x = 2 + \frac{3i}{n}.$$

The approximation of the area obtained by summing the areas of n rectangles with base Δx and heights $f(x_i)$ as i ranges from 1 to n is then given by

$$R_n = \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} \sqrt{x_i}\frac{3i}{n} = \sum_{i=1}^{n} \left(\sqrt{2 + \frac{3i}{n}}\right)\frac{3i}{n}.$$

The area under the curve is then $\lim_{n\to\infty} R_n$. That is, the area as the limit of a sum using the right endpoint method is

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\sqrt{2 + \frac{3i}{n}}\right)\frac{3i}{n}.$$

A diagram of the region, and an example rectangle, is given below: