Quiz 7 **Solutions**, Math 111, Section 2 (Vinroot)

For each of the following limits, explain what type of indeterminate form it is (at several stages if necessary) and evaluate. Show all steps.

(a): $\lim_{x\to\infty} \frac{\ln(x^2)}{x^{1/4}}$.

Solution: First, $\lim_{x\to\infty} \ln(x^2) = \infty$ and $\lim_{x\to\infty} x^{1/4} = \infty$, so this limit is an indeterminate form of type "$\frac{\infty}{\infty}$", and so we may apply L'Hospital's rule. We then have

$$\lim_{x\to\infty} \frac{\ln(x^2)}{x^{1/4}} = \lim_{x\to\infty} \frac{\frac{1}{x^2}\cdot 2x}{\frac{1}{4}x^{-3/4}} = \lim_{x\to\infty} \frac{8}{x^{\frac{1}{4}}} = 0.$$

(b): $\lim_{x\to 0^+} (3x)^x$.

Solution: We have $\lim_{x\to 0^+} 3x = 0$ and $\lim_{x\to 0^+} x = 0$, so this limit is an indeterminate form of type "0^0". We let $y = (3x)^x$, and consider $\ln y = x\ln(3x)$. Now, considering the limit

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} x\ln(3x),$$

since $\lim_{x\to 0^+} x = 0$ and $\lim_{x\to 0^+} \ln(3x) = -\infty$, this limit is an indeterminate form of type "$0\cdot\infty$". Re-writing this product as a quotient, we have:

$$\lim_{x\to 0^+} x\ln(3x) = \lim_{x\to 0^+} \frac{\ln(3x)}{1/x},$$

which is an indeterminate form of type "$\frac{\infty}{\infty}$", so we may apply L'Hospital's rule. So, we have

$$\lim_{x\to 0^+} \frac{\ln(3x)}{1/x} = \lim_{x\to 0^+} \frac{\frac{3}{3x}}{-x^{-2}} = \lim_{x\to 0^+} (-x) = 0.$$

Since we now have $\lim_{x\to 0^+} \ln y = 0$, then we have (by continuity of ln), $\ln(\lim_{x\to 0^+} y) = 0$. Thus, $\lim_{x\to 0^+} y = e^0 = 1$, that is,

$$\lim_{x\to 0^+} (3x)^x = 1.$$