Quiz 7 Solutions, Math 111, Section 2 (Vinroot)

For each of the following limits, explain what type of indeterminate form it is (at several stages if necessary) and evaluate. Show all steps.

(a): \( \lim_{x \to \infty} \frac{\ln(x^2)}{x^{1/4}} \).

**Solution:** First, \( \lim_{x \to \infty} \ln(x^2) = \infty \) and \( \lim_{x \to \infty} x^{1/4} = \infty \), so this limit is an indeterminate form of type \( \frac{\infty}{\infty} \), and so we may apply L’Hospital’s rule. We then have

\[
\lim_{x \to \infty} \frac{\ln(x^2)}{x^{1/4}} = \lim_{x \to \infty} \frac{\frac{1}{x} \cdot 2x}{\frac{1}{4} x^{-3/4}} = \lim_{x \to \infty} \frac{8}{x^{3/4}} = 0.
\]

(b): \( \lim_{x \to 0^+} (3x)^x \).

**Solution:** We have \( \lim_{x \to 0^+} 3x = 0 \) and \( \lim_{x \to 0^+} x = 0 \), so this limit is an indeterminate form of type \( “0^0” \). We let \( y = (3x)^x \), and consider \( \ln y = x \ln(3x) \). Now, considering the limit

\[
\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} x \ln(3x),
\]

since \( \lim_{x \to 0^+} x = 0 \) and \( \lim_{x \to 0^+} \ln(3x) = -\infty \), this limit is an indeterminate form of type \( “0 \cdot \infty” \). Re-writing this product as a quotient, we have:

\[
\lim_{x \to 0^+} x \ln(3x) = \lim_{x \to 0^+} \frac{\ln(3x)}{1/x},
\]

which is an indeterminate form of type \( \frac{0}{\infty} \), so we may apply L’Hospital’s rule. So, we have

\[
\lim_{x \to 0^+} \frac{\ln(3x)}{1/x} = \lim_{x \to 0^+} \frac{3x}{-x^2} = \lim_{x \to 0^+} (-x) = 0.
\]

Since we now have \( \lim_{x \to 0^+} \ln y = 0 \), then we have (by continuity of \( \ln \)), \( \ln(\lim_{x \to 0^+} y) = 0 \). Thus, \( \lim_{x \to 0^+} y = e^0 = 1 \), that is,

\[
\lim_{x \to 0^+} (3x)^x = 1.
\]