Quiz 4 **Solutions**, Math 111, Section 2 (Vinroot)

As always, show all steps clearly in your solution.

(a): Compute $\frac{dy}{dx}$ if $x^2y^5 + y^3 = \cos(x^2)$.

Solution: Using implicit differentiation, we take the derivative with respect to x of both sides, after which we must apply the product rule in the first term on the left side, and the chain rule on all terms. We then gather all terms including $y' = dy/dx$ on one side in order to solve for it:

$$\begin{aligned}
\frac{d}{dx}(x^2y^5) + \frac{d}{dx}(y^3) =& \frac{d}{dx}(\cos(x^2)) \\
x^2\frac{d}{dx}(y^5) + 2xy^5 + 3y^2y' =& -\sin(x^2)2x \\
x^2(5y^4y') + 2xy^5 + 3y^2y' =& -2x\sin(x^2) \\
y'(5x^2y^4 + 3y^2) =& -2x\sin(x^2) - 2xy^5 \\
y' =& \frac{-2x\sin(x^2) - 2xy^5}{5x^2y^4 + 3y^2}
\end{aligned}$$

(b): Compute y' if $y = (2x^2 + 1)^{\sin(x)}$ (Hint: First take ln of both sides).

Solution: The idea is to use logarithmic differentiation, since there is a function in the exponent of the function we want to differentiate. Taking the natural logarithm of both sides gives:

$$\ln(y) = \sin(x)\ln(2x^2 + 1).$$

We now differentiate both sides with respect to x, where we use the fact that $\frac{d}{dx}(\ln(y)) = \frac{y'}{y}$, and we must use the product and chain rule on the right as well. We obtain:

$$\begin{aligned}
\frac{d}{dx}(\ln(y)) &= \frac{d}{dx}(\sin(x)\ln(2x^2 + 1)) \\
\frac{y'}{y} &= \ln(2x^2 + 1)\frac{d}{dx}(\sin(x)) + \sin(x)\frac{d}{dx}(\ln(2x^2 + 1) \\
\frac{y'}{y} &= \ln(2x^2 + 1)\cos(x) + \sin(x)\frac{4x}{2x^2 + 1} \\
y' &= y\left(\ln(2x^2 + 1)\cos(x) + \sin(x)\frac{4x}{2x^2 + 1}\right) \\
y' &= (2x^2 + 1)^{\sin(x)}\left(\ln(2x^2 + 1)\cos(x) + \sin(x)\frac{4x}{2x^2 + 1}\right)
\end{aligned}$$