Quiz 3 Solutions Math 111, Section 2 (Vinroot)

(a): Compute $\frac{dy}{dx}$ if $y = x^2 \sec(e^x)$.

Solution: First, by the product rule for derivatives, we have
\[
\frac{dy}{dx} = x^2 \frac{d}{dx} (\sec(e^x)) + \sec(e^x) \frac{d}{dx} (x^2)
\]
\[
= x^2 \frac{d}{dx} (\sec(e^x)) + 2x \sec(e^x)
\]
Next, if we write $f(x) = \sec(x)$ and $g(x) = e^x$, then $\sec(e^x) = f(g(x))$. Then $f'(x) = \sec(x) \tan(x)$ and $g'(x) = e^x$. By the Chain rule, then, we have
\[
\frac{d}{dx} (\sec(e^x)) = f'(g(x))g'(x)
\]
\[
= \sec(e^x) \tan(e^x)e^x.
\]
In the end, we then have
\[
\frac{dy}{dx} = x^2 e^x \sec(e^x) \tan(e^x) + 2x \sec(e^x).
\]

(b): Suppose $h(x) = -\cos(f(x))$, where $f$ is a differentiable function such that $f(1) = \pi/2$ and $f'(1) = 3$. Compute $h'(1)$.

Solution: If we write $g(x) = -\cos(x)$, then $h(x) = g(f(x))$, where $g'(x) = -(-\sin(x)) = \sin(x)$. By the Chain rule, we have
\[
h'(x) = g'(f(x))f'(x) = \sin(f(x))f'(x).
\]
To find $h'(1)$, we substitute in $x = 1$, and we have
\[
h'(1) = \sin(f(1))f'(1) = \sin(\pi/2) \cdot 3 = 1 \cdot 3 = 3.
\]
So $h'(1) = 3$. 