Quiz 3 **Solutions** Math 111, Section 2 (Vinroot)

(a): Compute $\frac{dy}{dx}$ if $y = x^2 \sec(e^x)$.

Solution: First, by the product rule for derivatives, we have

$$\begin{aligned}\frac{dy}{dx} &= x^2 \frac{d}{dx}(\sec(e^x)) + \sec(e^x)\frac{d}{dx}(x^2) \\ &= x^2 \frac{d}{dx}(\sec(e^x)) + 2x\sec(e^x)\end{aligned}$$

Next, if we write $f(x) = \sec(x)$ and $g(x) = e^x$, then $\sec(e^x) = f(g(x))$. Then $f'(x) = \sec(x)\tan(x)$ and $g'(x) = e^x$. By the Chain rule, then, we have

$$\begin{aligned}\frac{d}{dx}(\sec(e^x)) &= f'(g(x))g'(x) \\ &= \sec(e^x)\tan(e^x)e^x.\end{aligned}$$

In the end, we then have

$$\frac{dy}{dx} = x^2 e^x \sec(e^x)\tan(e^x) + 2x\sec(e^x).$$

(b): Suppose $h(x) = -\cos(f(x))$, where f is a differentiable function such that $f(1) = \pi/2$ and $f'(1) = 3$. Compute $h'(1)$.

Solution: If we write $g(x) = -\cos(x)$, then $h(x) = g(f(x))$, where $g'(x) = -(-\sin(x)) = \sin(x)$. By the Chain rule, we have

$$h'(x) = g'(f(x))f'(x) = \sin(f(x))f'(x).$$

To find $h'(1)$, we substitute in $x = 1$, and we have

$$h'(1) = \sin(f(1))f'(1) = \sin(\pi/2)\cdot 3 = 1\cdot 3 = 3.$$

So $h'(1) = 3$.