Quiz 2 **Solutions**, Math 111, Section 2 (Vinroot)

Use the **definition** of the derivative to compute $f'(x)$ if $f(x) = \frac{1}{x+1}$. At what values is f *not* differentiable? Show all steps in an organized way and give appropriate explanation where necessary.

Solution: By the definition of $f'(x)$, we have

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} \\ &= \lim_{h\to 0} \frac{\frac{(x+1)-(x+h+1)}{(x+h+1)(x+1)}}{h} \\ &= \lim_{h\to 0} \frac{-h}{h(x+h+1)(x+1)} \\ &= \lim_{h\to 0} \frac{-1}{(x+h+1)(x+1)} \qquad \text{(since } h \neq 0 \text{ as } h \to 0) \\ &= \frac{-1}{(x+1)^2}, \end{aligned}$$

since the function $\dfrac{-1}{(x+h+1)(x+1)}$ is a continuous on its domain, as a function of h. We have f is not differentiable at $x = -1$, since f is not continuous at $x = -1$ (it is not defined there). But, $f'(x) = \frac{-1}{(x+1)^2}$ is defined for all $x \neq -1$, so f is not differentiable only for $x = -1$.