Quiz 2 Solutions, Math 111, Section 2 (Vinroot)

Use the definition of the derivative to compute \( f'(x) \) if \( f(x) = \frac{1}{x+1} \). At what values is \( f \) not differentiable? Show all steps in an organized way and give appropriate explanation where necessary.

Solution: By the definition of \( f'(x) \), we have

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
= \lim_{h \to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}
= \lim_{h \to 0} \frac{(x+1)-(x+h+1)}{h(x+h+1)(x+1)}
= \lim_{h \to 0} \frac{-h}{h(x+h+1)(x+1)}
= \lim_{h \to 0} \frac{-1}{(x+h+1)(x+1)} \quad \text{(since } h \neq 0 \text{ as } h \to 0) 
= \frac{-1}{(x+1)^2},
\]

since the function \( \frac{-1}{(x+h+1)(x+1)} \) is a continuous on its domain, as a function of \( h \). We have \( f \) is not differentiable at \( x = -1 \), since \( f \) is not continuous at \( x = -1 \) (it is not defined there). But, \( f'(x) = \frac{-1}{(x+1)^2} \) is defined for all \( x \neq -1 \), so \( f \) is not differentiable only for \( x = -1 \).