Quiz 1 **Solutions**, Math 111, Section 2 (Vinroot)

Find values a and b such that the following function is continuous at all real numbers. Give an explanation of your solution, using the definition of continuous.

$$f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ ax + b & \text{if } -1 \leq x \leq 1 \\ \sqrt{x+3} & \text{if } x \geq 1. \end{cases}$$

Solution: From the definition of $f(x)$, since x^2, any linear function $ax+b$, and $\sqrt{x+3}$ for $x \geq -3$ are all continuous, then f is already continuous for all $x \neq -1, 1$. To force f to be continuous at $x = -1$ and $x = 1$, we need, by the definition of being continuous,

$$\lim_{x \to -1} f(x) = f(-1) = -a + b \quad \text{and} \quad \lim_{x \to 1} f(x) = f(1) = a + b.$$

From the definition of f, we have

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} x^2 = (-1)^2 = 1 \quad \text{and} \quad \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \sqrt{x+3} = \sqrt{1+3} = 2.$$

So, for f to be continuous at $x = -1$ and $x = 1$, we need

$$\lim_{x \to -1^-} f(x) = f(-1) = -a + b \quad \text{and} \quad \lim_{x \to 1^+} f(x) = f(1) = a + b.$$

Therefore, we need $-a + b = 1$ and $a + b = 2$. Adding these together gives $2b = 3$, so $b = 3/2$, and subbing this value of b into either equation gives $a = 1/2$. So the values of a and b must be $a = 1/2$ and $b = 3/2$.