Quiz 0 **Solutions**, Math 111, Section 2 (Vinroot)

(a): Evaluate the following, with an explanation. That is, show and give a brief explanation of the main steps:

$$\lim_{x\to 2} \frac{x^2 - 3x + 2}{x^2 - 4}$$

Solution: First note that the denominator is $x^2 - 4 = (x-2)(x+2)$, which is 0 at $x = 2$. So, we cannot simply substitute in $x = 2$. Now, by factoring,

$$\lim_{x\to 2} \frac{x^2 - 3x + 2}{x^2 - 4} = \lim_{x\to 2} \frac{(x-1)(x-2)}{(x+2)(x-2)} = \lim_{x\to 2} \frac{x-1}{x+2},$$

where we can cancel the common factor $(x-2)$, since in the limit as $x \to 2$, we have $x \neq 2$, so that $x - 2 \neq 0$. Finally, by limit laws, we have

$$\lim_{x\to 2} \frac{x-1}{x+2} = \frac{\lim_{x\to 2}(x-1)}{\lim_{x\to 2}(x+2)} = \frac{2-1}{2+2} = \frac{1}{4},$$

where we may use the quotient limit law since $\lim_{x\to 2}(x+2) = 4 \neq 0$.

(b): State whether the following is "TRUE" or "FALSE", with a careful explanation:

$$\lim_{x\to \pi/2^-} \frac{x^2}{\cos x} = \infty$$

Solution: This statement is TRUE. Recall that $\cos(\pi/2) = 0$, and when $x < \pi/2$ but $x > 0$, then $\cos x > 0$. So, as $x \to \pi/2^-$, $\cos x \to 0$ and $\cos x$ is positive, so $1/\cos x \to \infty$. Since $x^2 \to \pi^2/4$ as $x \to \pi/2^-$, then the numerator does not affect the limit getting large. Thus, we have

$$\lim_{x\to \pi/2^-} \frac{x^2}{\cos x} = \infty,$$

as claimed.