1. Use polynomial division to write the following rational expression as the quotient plus a simplified rational expression:

\[
\frac{x^4 + x + 2}{x^2 + 1}
\]

Solution:

\[
\begin{array}{c}
\begin{array}{c}
\underline{x^2 - 1} \\
\underline{x^2 + 1}
\end{array} \\
x^4 + 0x^3 + 0x^2 + x + 2 \\
- (x^4 + x^2) \\
- \underline{x^2 + x + 2} \\
- (-x^2 - 1) \\
\hline
x + 3 \quad \text{remainder}
\end{array}
\]

So

\[
\frac{x^4 + x + 2}{x^2 + 1} = \frac{x^2 - 1 + \frac{x + 3}{x^2 + 1}}
\]

2. Briefly explain why \(x^2 + 2x + 2 = 0\) has no real solutions, and why this means the expression \(\frac{1}{x^2 + 2x + 2}\) is defined for all real values of \(x\).

Solution: In the quadratic formula with \(a=1, b=2, c=2\), we have \(b^2 - 4ac = 4 - 4(1)(2) = -4 < 0\), which means \(x^2 + 2x + 2 = 0\) has no real solutions.

\(\frac{1}{x^2 + 2x + 2}\) is only undefined if the denominator is 0. Since \(x^2 + 2x + 2 = 0\) never holds for real values \(x\), then \(\frac{1}{x^2 + 2x + 2}\) is defined for all real values of \(x\).