Quiz 2 **Solutions**, Math 103 - Precalculus

1. Use polynomial division to write the following rational expression as the quotient plus a simplified rational expression:

$$\frac{x^4+x+2}{x^2+1}$$

Solution:

$$
\begin{array}{r l}
 & x^2 - 1 \\
x^2+1 \,\big)\!\! & \overline{x^4 + 0x^3 + 0x^2 + x + 2} \\
 & -(x^4 \qquad\quad + x^2) \\ \hline
 & \qquad\quad -x^2 + x + 2 \\
 & \qquad -(-x^2 \quad\ - 1) \\ \hline
 & \qquad\qquad\quad x + 3 \leftarrow \text{remainder}
\end{array}
$$

So $\dfrac{x^4+x+2}{x^2+1} = \boxed{x^2 - 1 + \dfrac{x+3}{x^2+1}}$

2. Briefly explain why $x^2+2x+2=0$ has no real solutions, and why this means the expression $\dfrac{1}{x^2+2x+2}$ is defined for all real values of x.

Solution: In the quadratic formula with $a=1$, $b=2$, $c=2$, we have $b^2-4ac = 4-4(1)(2) = -4 < 0$, which means $x^2+2x+2=0$ has no real solutions.

$\dfrac{1}{x^2+2x+2}$ is only undefined if the denominator is 0.

Since $x^2+2x+2=0$ never holds for real values x, then $\dfrac{1}{x^2+2x+2}$ is defined for all real values of x.