

Homework #8 Part B

Please note that this is not to be turned in.

The inverse trigonometric functions have restricted ranges, so that they are actual functions:

(i): $y = \arcsin(x) = \sin^{-1}(x)$ has domain $-1 \leq x \leq 1$, and range $-\pi/2 \leq y \leq \pi/2$,

(ii): $y = \arccos(x) = \cos^{-1}(x)$ has domain $-1 \leq x \leq 1$, and range $0 \leq y \leq \pi$, and

(iii): $y = \arctan(x) = \tan^{-1}(x)$ has domain all real numbers and range $-\pi/2 < y < \pi/2$.

1. Evaluate each of the following:

(a): $\arcsin(-\sqrt{3}/2)$ (b): $\arccos(\sqrt{2}/2)$ (c): $\arctan(-1)$

(d): $\arcsin(1/2)$ (e): $\arccos(-1)$ (f): $\arctan(\sqrt{3}/3)$

2. Evaluate each of the following:

(a): $\cos(\arcsin(3/5))$ (b): $\tan(\arccos(1/3))$

3. If $-1 \leq x \leq 1$, explain why $\sin(\arccos(x)) = \sqrt{1-x^2} = \cos(\arcsin(x))$

4. Graph each of the following, and give the domain and range of each:

(a): $f(x) = \arcsin(x) + \frac{\pi}{2}$ (b): $g(x) = \arccos(x-1) + \pi$ (c): $h(x) = \arctan(x+1) - \frac{\pi}{4}$

The following are some review trigonometry problems:

5. If $\frac{\sin^2(\alpha)}{\cos^2(\alpha)} = \frac{16}{25}$, and $\pi/2 < \alpha < \pi$, what is the value of $\cot(\alpha)$? Explain.

6. Using the angle sum identities for sine and cosine, $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$ and $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$, obtain the following identity (the angle sum identity for tangent):

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}.$$

7. Find all values of x such that

$$2\cos^3(x) - 4\cos^2(x) - \cos(x) + 2 = 0.$$

Hint: Let $w = \cos(x)$, then factor the polynomial in w by first using synthetic division.