Math 103 Precalculus (Vinroot)

November 16, 2015

Due: Monday, November 23, 2015

Homework #7 Part A

1. Evaluate each of the following (this is review from HW #6):

(a): $\sin(5\pi/3)$ **(b):** $\csc(11\pi/6)$ **(c):** $\cot(7\pi)$

(d): $\tan(\pi/3)$ **(e):** $\cos(21\pi/2)$ **(f):** $\sec(-7\pi/6)$

In problems **2-4**, obtain each of the trigonometric identities:

2. $\tan(x) + \cot(x) = \sec(x)\csc(x)$

3. $\sin^2(\theta)\csc^2(\theta) - \sin^2(\theta) = \cos^2(\theta)$

4. $\sin^4(x) + \cos^4(x) = 1 - 2\cos^2(x) + 2\cos^4(x)$

The *angle sum* identities for sine and cosine are the following, where α and β are two angles:

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) \quad \text{and} \quad \cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

5. Taking $\beta = \alpha$ in the angle sum identities, obtain the *double-angle* identities below:

(a): $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$

(b): $\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha) = 2\cos^2(\alpha) - 1 = 1 - 2\sin^2(\alpha)$

6. Use the double angle identities for cosine to obtain the *half-angle* identities:

(a): From $\cos(2\alpha) = 2\cos^2(\alpha) - 1$, solve for $\cos(\alpha)$ and let $\alpha = \theta/2$ to obtain

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos(\theta)}{2}}$$

(b): From $\cos(2\alpha) = 1 - 2\sin^2(\alpha)$, solve for $\sin(\alpha)$ and let $\alpha = \theta/2$ to obtain

$$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos(\theta)}{2}}$$

7. Use the half-angle identities to evaluate the following (simplify as much as possible):

(a): $\sin(\pi/8)$ **(b):** $\cos(\pi/12)$

November 18, 2015

Due: Monday, November 23, 2015

Homework #7 Part B

1. Evaluate each of the following (more review from HW #6):

(a): $\csc(7\pi/4)$ **(b):** $\cos(-2\pi/3)$ **(c):** $\cot(3\pi/2)$

(d): $\tan(4\pi/3)$ **(e):** $\sec(\pi/6)$ **(f):** $\sin(-5\pi/6)$

2. Obtain the following trigonometric identities:

(a): $\dfrac{1+\sin(x)}{\cos(x)} = \sec(x) + \tan(x)$ **(b):** $\dfrac{\cot(x) - \tan(x)}{\cot(x) + \tan(x)} = \cos(2x)$

3. Find all values of x which satisfy the following equations:

(a): $3\cos^2(x) - \cos(2x) = 1$ (**Hint:** First substitute $\cos(2x) = 2\cos^2(x) - 1$).

(b): $5\cos(3x) + 5\sin(x)\cos(3x) = 0$.

4. If θ is an angle such that $\cos(\theta) = \frac{2}{3}$, what are the values of $\sin(\theta), \tan(\theta), \sec(\theta), \csc(\theta)$, and $\cot(\theta)$?

5. **(a):** Draw a picture on the unit circle to show why $\sin\left(x - \frac{\pi}{2}\right) = -\cos(x)$ and $\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$.

(b): Use **(a)** to show that $-\tan\left(x - \frac{\pi}{2}\right) = \cot(x)$ and $\sec\left(x - \frac{\pi}{2}\right) = \csc(x)$.

(c): Use **(b)** and the graphs of $y = \tan(x)$ and $y = \sec(x)$ to sketch the graphs of $y = \cot(x)$ and $y = \csc(x)$.

6. A regular n-gon is inscribed a circle of radius 1. Find that the area of the n-gon is exactly $\frac{n}{2}\sin\left(\frac{2\pi}{n}\right)$.

Hint: Draw n triangles which meet in the center of the n-gon, so that each triangle has an angle of measure $2\pi/n$. Find the area of one of these triangles, and multiply by n.