Homework #5 Part A

1. Find the domain and range of the function \( h(x) = \ln |x| \), and explain.

2. Find the domain and range of the function \( f(x) = e^{x^2} \), and explain.

3. If the domain of \( g(x) \) is the set of all positive real numbers (so all \( x > 0 \)), what is the domain of \( g(|x|) \)? What is the domain of \( g(x^2) \)? Explain.

In problems 4 through 6, find (and write as simply as possible) the expressions given by the compositions \( (a) \) \( f(g(x)) \), \( (b) \) \( g(f(x)) \), and \( (c) \) \( f(f(x)) \), where the functions \( f(x) \) and \( g(x) \) are given.

4. \( f(x) = \sqrt{x}, g(x) = x^2 \) (Be careful with \( f(g(x)) \)). Is \( \sqrt{x^2} = x \) for all \( x \)?

5. \( f(x) = x^2 + x + 1, g(x) = 3^x \).

6. \( f(x) = e^x, g(x) = \sqrt{x} \). Also, if \( h(x) = x/2 \), explain why \( g(f(x)) = f(h(x)) \).

7. Find the \( x \) and \( y \)-intercepts of each of the following functions:
   \( (a) \): \( f(x) = \ln(x^2 - 8x + 13) \)
   \( (b) \): \( g(x) = e^{6x} - 3e^{3x} - 10 \) (Hint: \( e^{6x} = (e^{3x})^2 \), so factor this like a quadratic for the \( x \)-intercept).

8. For each \( f(x) \) given, compute and simplify the expression \( \frac{f(x + h) - f(x)}{h} \)
   \( (a) \): \( f(x) = x^2 + 5x \)
   \( (b) \): \( f(x) = \frac{1}{x} \)
Homework #5 Part B

1. (a): Sketch a graph of the functions $y = x^2 + 3$, $y = 2^x + 1$, and $y = \ln(x) - 3$, using the graphs of $y = x^2$, $y = 2^x$, and $y = \ln(x)$.
   (b): Given a real number $c$ and a function $y = f(x)$, how are the graphs of $y = f(x)$ and $y = f(x) + c$ related?

2. (a): Sketch a graph of the functions $y = (x - 3)^2$, $y = 2^{x-1}$, and $y = \ln(x + 3)$, using the graphs of $y = x^2$, $y = 2^x$, and $y = \ln(x)$.
   (b): Given a real number $a$ and a function $y = g(x)$, how are the graphs of $y = g(x)$ and $y = g(x - a)$ related?

3. Find the points of intersection of each of the following pairs of graphs, and sketch the graphs.
   (a): $y = x - 3$ and $(x - 3)^2 + (y + 3)^2 = 9$
   (b): $y = x^2 + 2$ and $y = 3x + 1$

4. Find the equation of the circle of radius 2 and center $(-3, 1)$, and sketch the graph. Find the equations of two functions $y = f(x)$ and $y = g(x)$ whose graphs together give the graph of this circle.

5. The function $y = f(x)$ is even if $f(-x) = f(x)$ for every $x$ and the function $y = g(x)$ is odd if $g(-x) = -g(x)$ for every $x$.
   (a): Show that $f(x) = x^4$, $f(x) = |x|$, and $f(x) = 1/x^2$ are each even. Sketch each graph.
   (b): Show that $g(x) = x^3$, $g(x) = \sqrt[3]{x}$, and $g(x) = 1/x$ are each odd. Sketch each graph.
   (c): What can you say about the graph of an even function and the graph of an odd function?

6. Sketch a graph of each of the following:
   (a): $y = |2x + 1|
   (b): y = |\ln(x)|$
   (c): $y = \ln |x|
   (d): |x| + |y| = 1