

Final Review Problems

These are problems to prepare for the Math 103 Final, which will be on Wednesday, December 9, 9:00 AM-12:00 Noon.

1. Solve for x in each of the following equations, or explain why there are no solutions:

(a): $2x^2 + 5x - 12 = 0$ (b): $x^2 + 3x - 3 = 0$, by completing the square.

(c): $x^2 + 3x + 3 = 0$ (d): $2x^3 - x^2 - 2x + 1 = 0$ (e): $\cos^2(2x) - 2\cos(2x) + 1 = 0$

(f): $\ln(x^2) - \ln(x + 3) = \ln(2)$ (g): $(2e^x + A)\cos(2yb) - e^x \sin(Aby) = ze^x - \ln(\arctan(2zb))$

2. Evaluate each of the following (your final answer should be a number):

(a): $\log_4(\frac{1}{8})$ (b): $27^{-2/3}$ (c): $13^{\frac{\log_7(3)}{\log_7(13)}}$ (d): $\ln((e^2 e^{1/2})^{-2/3})$

(e): $3^{-1/2} (\frac{1}{27})^{-1/6}$ (f): $3^{\log_{28}(34^{\ln 1})}$ (g): $\tan(13\pi/6)$ (h): $\csc(5\pi/4)$

(i): $\sin(-7\pi/3)$ (j): $\arctan(0)$ (k): $\arcsin(-\sqrt{2}/2)$ (l): $\sin(\arccos(1/6))$

3. (a): One cone has $1/5$ of the height and four times the base radius of a second cone. Find the ratio of the volume of the first cone to the volume of the second cone.

(b): One right triangle has base 4 and hypotenuse 6, and a similar triangle has hypotenuse 12. What is the area of the larger triangle?

4. Obtain the following trigonometric identities:

(a): $\frac{\cot^2(\alpha)}{\csc(\alpha)} = \csc(\alpha) - \sin(\alpha)$ (b): $\cot^2(x) \tan^2(x) \sin^2(x) + (\cos(x) \csc(x) \sin(x))^2 = 1$

5. Graph each of the following functions, then find the inverse function and graph the inverse function:

(a): $f(x) = 5x - 3$ (b): $g(x) = 3^x - 2$ (c): $h(x) = \sqrt[3]{x} + 2$

6. Find the x -intercepts, y -intercepts, domain, and range of each of the following functions:

(a): $f(x) = \sqrt{x} - 8$ (b): $g(x) = \log_2(3x + 4)$

7. Find all values of x for which each of the expressions are undefined:

(a): $\log_7(\sqrt{x-5})$ (b): $\frac{17x^{10} - 13x^9 + 1}{\cos^2(x)}$

(c): $(x^2 - 3x - 10)^{1/4}$ (d): $\frac{17x^{10} - 13x^9 + 2}{(x^2 - 4)(x^2 + 2x + 5)}$

8. Simplify each of the following expressions in the way specified:

(a): Write the following as a polynomial plus a simplified rational expression: $\frac{x^4 + 3x^2 - x + 2}{x^2 - x + 2}$.

(b): Write the following as a single logarithm: $2\log_3(x) - \log_3(x+3) - \frac{\ln(x^2)}{\ln 3}$.

(c): Write the following as a single trigonometric function: $\frac{\tan(x)\sin^2(x)\cot^2(x)\sec^2(x)}{\csc^2(x)\tan^2(x)\cos^2(x)}$.

(d): Write the following as a single power of $(x^2 + 2)$:

$$\frac{(x^2 + 2)^4}{\sqrt[3]{x^2 + 2}} (\sqrt{(x^2 + 2)^{-2}}) (x^2 + 2)^{2/3}.$$

9. Sketch a graph of each of the following:

(a): $(x-2)^2 + (y+3)^2 = 4$ (b): $y = \sin(x + \frac{\pi}{2}) + 1$ (c): $y = |x^2 - 5|$

10. (a): Find the points of intersection of $y = x^2 - 3$ and $y = 2x + 5$, and sketch a graph.

(b): Find the point of intersection of $y = \frac{-1}{\ln(x)}$ and $y = \ln(x) - 2$

11. Suppose $f(x) = x^3$ and g is a function with an inverse such that $g(2) = -1$. Evaluate the following: (a): $f(g(2))$ (b): $f^{-1}(-8)$ (c): $f(g^{-1}(-1))$ (c): $f(f^{-1}(213))$

12. Find the inverse function $f^{-1}(x)$ for each of the following:

(a): $f(x) = \log_3(x^3 - 5)$ (b): $f(x) = \frac{1}{e^{x^5} + 2}$

13. (a): Find the equation of the line going through the point $(1, -1)$ and which is perpendicular to the line going through the points $(2, 3)$ and $(-1, 0)$. Sketch a graph of both lines.

(b): Find all values of x such that $2x^2 - 11x + 12 > 0$. Use this to sketch a graph of $y = 2x^2 - 11x + 12$.

14. Let $f(x) = \sqrt{x^2 + 2}$ and $g(x) = \cos(x^2)$.

(a): Find the domain and range of $f(x)$ and $g(x)$.

(b): Write down (in simplest form) the functions $f(g(x))$ and $g(f(x))$.

(c): Do $f(x)$ or $g(x)$ have inverse functions? Why or why not?