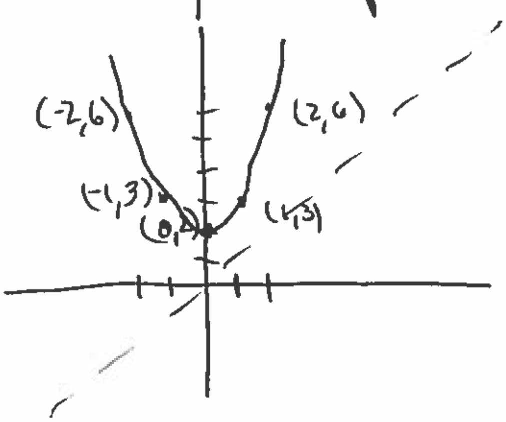
# Math 103 - HW#8A

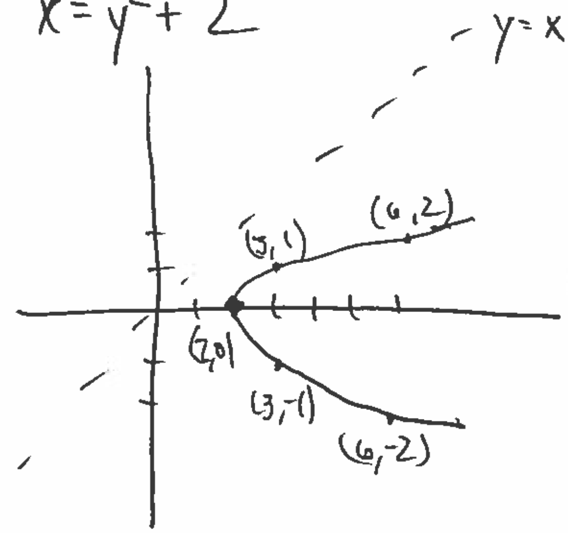
A1. For each of these, we just take the inverse equation by switching x and y (we are not solving for y, there is not an inverse function for these). We can graph each inverse equation by reflecting through the line $y=x$, or by plotting points (for one, then switching coordinates for the other)

(a) $y = x^2 + 2$

(shift $y=x^2$ up two units)

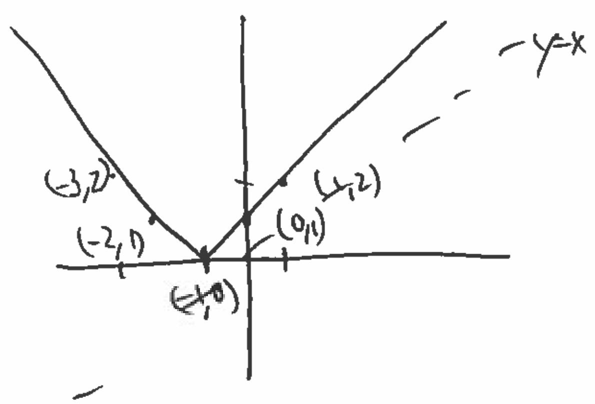



$x = y^2 + 2$

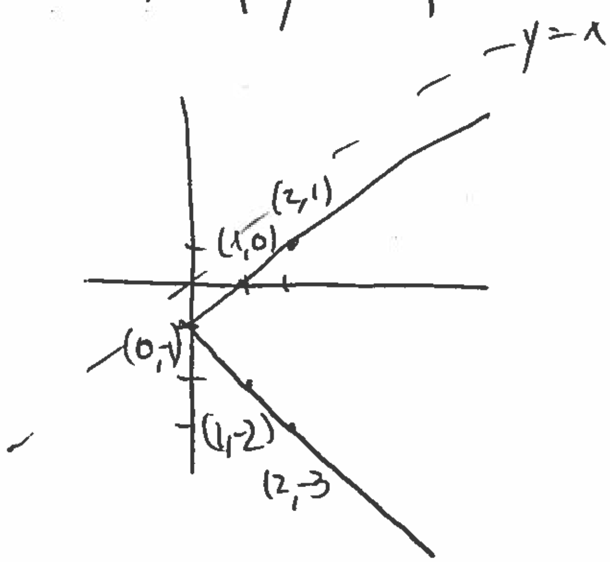



(b) $y = |x+1|$

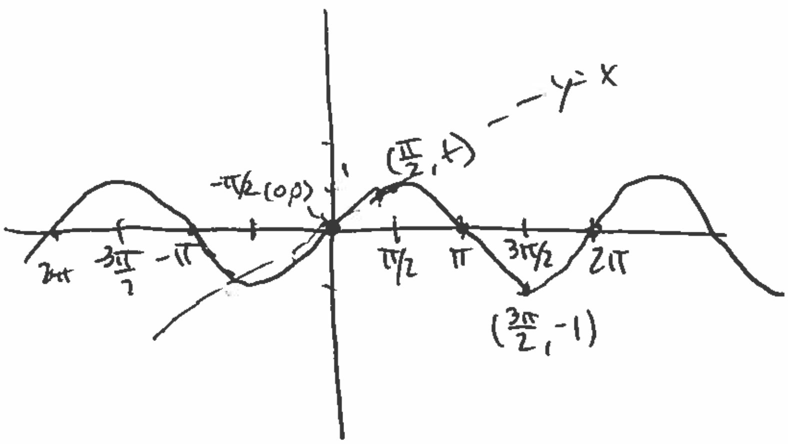
(shift $y=|x|$ one unit to the left)



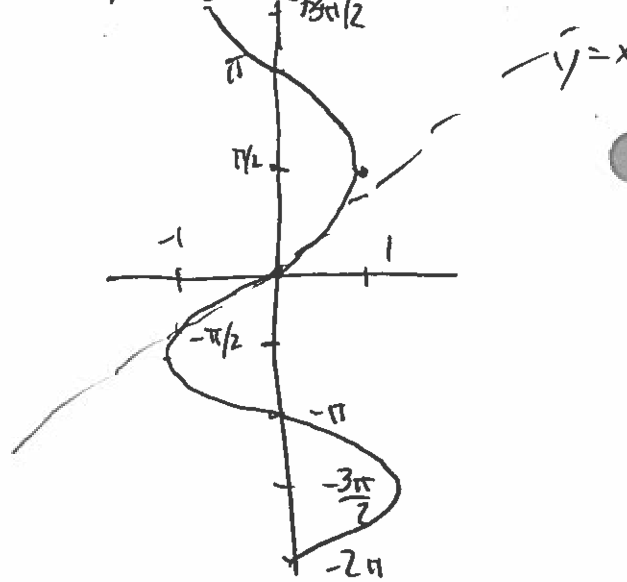

$x = |y+1|$



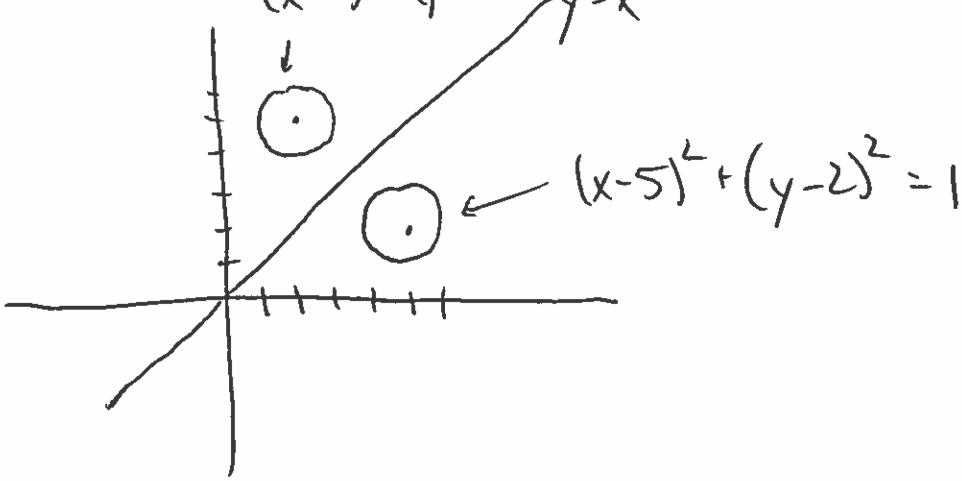
A1 (c) $y = \sin(x)$




$x = \sin(y)$




A2. The equation of a circle with radius $r$ and center $(a,b)$ is $(x-a)^2+(y-b)^2=r^2$. The inverse equation is obtained by switching $x$ and $y$, which gives $(y-a)^2+(x-b)^2=r^2$, or $(x-b)^2+(y-a)^2=r^2$. From the general form, $(x-b)^2+(y-a)^2=r^2$ is a circle of radius $r$ and center $(b,a)$. For $r=1$ and $(a,b)=(2,5)$ these two circles are graphed below:



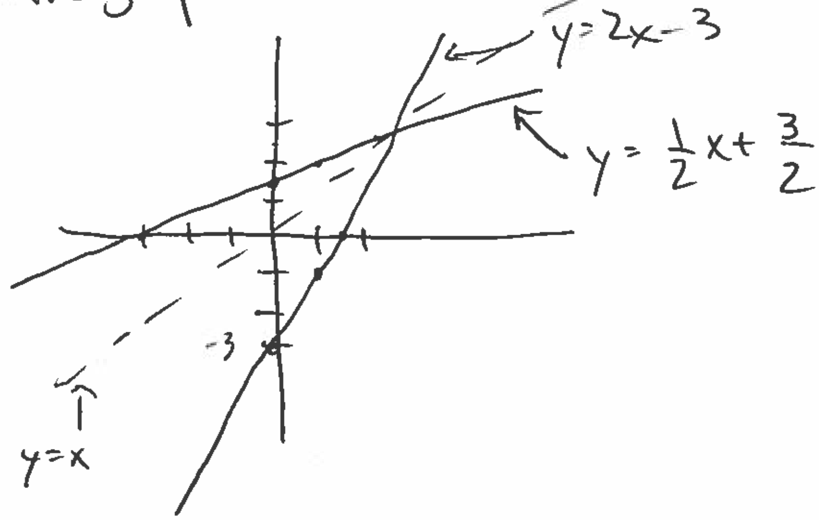
A3 This problem is like Problem A1, but we can solve for the inverse function for these.

(a) $f(x) = 2x - 3$, $y = 2x - 3$, so inverse equation is $x = 2y - 3$. Solving for $y$,

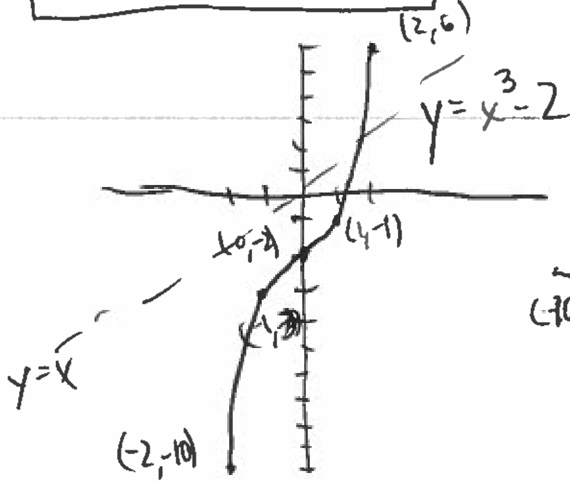
$$x + 3 = 2y, \text{ so } y = \frac{1}{2}x + \frac{3}{2}.$$

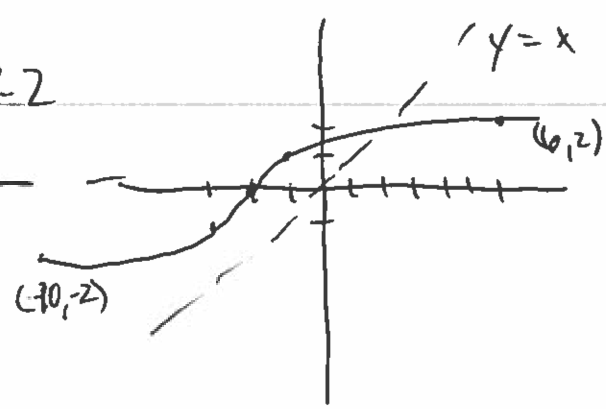
So $\boxed{f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}}$

Both graphs are below:



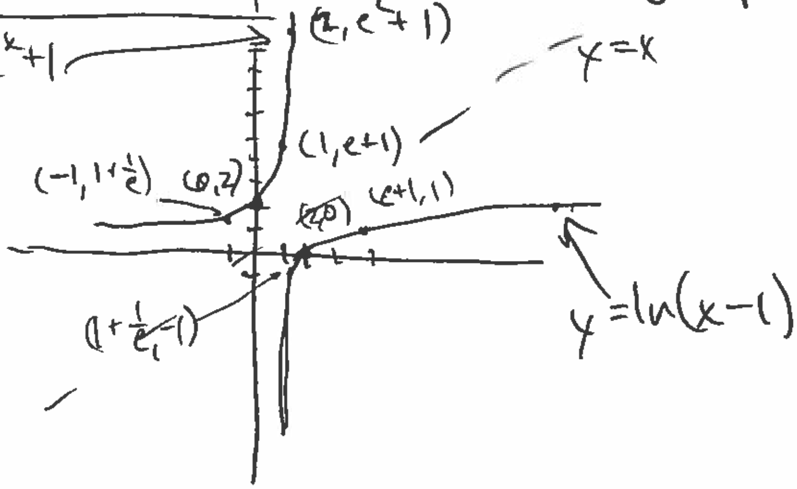

(b) $f(x) = x^3 - 2$, $y = x^3 - 2$, so inverse equation is $x + 2 = y^3$; solving for $y$ gives ~~$y = x$~~ $y = (x+2)^{1/3}$, so $\boxed{f^{-1}(x) = (x+2)^{1/3}}$. These are graphed below:



A3 (c): $f(x) = e^x + 1$, so $y = e^x + 1$, so inverse equation is $x = e^y + 1$. Solving for $y$, $x - 1 = e^y$, so $\ln(x-1) = \ln(e^y) = y$. So $y = \ln(x-1)$, or

$\boxed{f^{-1}(x) = \ln(x-1)}$. These are graphed below:




A4 (a) $y = e^{x^3} + 1$, so inverse equation is $x = e^{y^3} + 1$,

Solving for $y$, $x - 1 = e^{y^3}$, so $\ln(x-1) = \ln(e^{y^3}) = y^3$.

So $y = (\ln(x-1))^{1/3}$, so $\boxed{f^{-1}(x) = (\ln(x-1))^{1/3}}$

(b) $y = [\ln(x^3 - 2)]^{1/5}$, so inverse equation is $x = [\ln(y^3 - 2)]^{1/5}$.

Solving for $y$ gives $x^5 = \ln(y^3 - 2)$, (raising both sides to 5th power)

so $e^{x^5} = e^{\ln(y^3-2)} = y^3 - 2$.

So $y^3 = e^{x^5} + 2$, so $y = (e^{x^5} + 2)^{1/3}$.

So $\boxed{f^{-1}(x) = (e^{x^5} + 2)^{1/3}}$

A4 (c) $y = \frac{1}{(e^x+1)^3}$, so the inverse equation is

$x = \frac{1}{(e^y+1)^3}$. Solving for $y$: $\frac{1}{x} = (e^y+1)^3$,

so $\left(\frac{1}{x}\right)^{1/3} = e^y + 1$, so $e^y = \left(\frac{1}{x}\right)^{1/3} - 1 = x^{-1/3} - 1$.

Then $\ln(e^y) = \ln(x^{-1/3} - 1)$, so $y = \ln(x^{-1/3} - 1)$.

$\boxed{f^{-1}(x) = \ln(x^{-1/3} - 1)}$.

A5 (a) Since $f(1) = 2$, then $\boxed{f^{-1}(2) = 1}$.

(b) $g(1) = 0$, so $g^{-1}(0) = 1$. Now $f(g^{-1}(0)) =$
$= f(1) = \boxed{2}$.

(c) $f^{-1}(f(1)) = f^{-1}(2) = \boxed{1}$.

(d) $f^{-1}(g(4) - 5) = f^{-1}(3-5) = f^{-1}(-2) = 3$, since
$f(3) = -2$. So $f^{-1}(g(4) - 5) = \boxed{3}$.

(e) $g^{-1}(f^{-1}(-2)) = g^{-1}(3) = \boxed{4}$, since $g(4) = 3$.

(f) If $f(8) = y$, then $f^{-1}(y) = 8$. So $f^{-1}(f(8)) = f^{-1}(y) = \boxed{8}$

(g) $f^{-1}(-f(g^{-1}(3) - 1)) = f^{-1}(-f(4-1)) = f^{-1}(-f(3)) =$
$= f^{-1}(-(-2)) = f^{-1}(2) = \boxed{1}$.