Math 103 - HW #4C

1. (a): $\log_2 6 = \frac{\log_4 6}{\log_4 2}$, but $\log_4 2 = \frac{1}{2}$ since $4^{1/2} = \sqrt{4} = 2$. So $\log_2 6 = \frac{\log_4 6}{\frac{1}{2}} = 2\log_4 6 = \log_4 6^2$
$= \boxed{\log_4 36}$

(b): $\log_3 7 - 2\log_3 2 = \log_3 7 - \log_3 2^2 = \log_3 \frac{7}{4} =$
$= \boxed{\frac{\ln(7/4)}{\ln 3}}$.

2. (a): $(\ln 5)(\log_5 e) = \cancel{\ln 5} \cdot \frac{\ln e}{\cancel{\ln 5}}$, since $\log_5 e = \frac{\log_e e}{\log_e 5}$. So $(\ln 5)(\log_5 e) = \ln e = \boxed{1}$.

(b): $\frac{\ln 3 + \ln 12}{\ln 12 - \ln 2} = \frac{\ln(3 \cdot 12)}{\ln(\frac{12}{2})} = \frac{\ln 36}{\ln 6} = \log_6 36 = \boxed{2}$.

(c): $\frac{\log_5(\sqrt[6]{27})}{\log_5 3} = \log_3(\sqrt[6]{27}) = \log_3(27^{1/6}) = \log_3((3^3)^{1/6})$
$= \log_3(3^{1/2}) = \boxed{\frac{1}{2}}$.

(d) $\frac{a}{b} = \frac{\ln 4}{\ln 5} = \log_5 4$, so $5^{a/b} = 5^{\log_5 4} = \boxed{4}$.

3. First, similar to 1(a), $\log_2 3 = \frac{\log_4 3}{\log_4 2}$, where $\log_4 2 = \frac{1}{2}$ since $4^{1/2} = 2$. Now

$$\log_2 3 - \log_4 5 = \frac{\log_4 3}{\log_4 2} - \log_4 5 = \frac{\log_4 3}{\frac{1}{2}} - \log_4 5 =$$

$$= 2\log_4 3 - \log_4 5 = \log_4(3^2) - \log_4 5 = \log_4 9 - \log_4 5$$

$$= \log_4\left(\frac{9}{4}\right).$$

4. Since $2\frac{\log_2 x}{\log_2 e} = 2\log_e x = 2\ln x = \ln(x^2)$, we can rewrite the equation as $\ln 2 + \ln(x^2) = \ln(2-3x)$, so $\ln(2x^2) = \ln(2-3x)$. To "undo" the ln, we raise e to the power given by each side, so

$$e^{\ln(2x^2)} = e^{\ln(2-3x)},$$ which gives $2x^2 = 2-3x$.

Now $2x^2 + 3x - 2 = 0$ can be factored, as

$$(2x-1)(x+2) = 0,$$ giving $x = \frac{1}{2}$ or $x = -2$.

However, in the original equation, $\log_2 x$ is <u>undefined</u> when $x = -2 \leq 0$. So $x = -2$ is not a solution. When $x = \frac{1}{2}$, everything in the original equation is still defined. So $\boxed{x = \frac{1}{2}}$ is the only solution

5. Taking ln of both sides gives

$$\ln A = \ln\left[P\left(1+\frac{r}{n}\right)^{nt}\right] = \ln P + \ln\left(1+\frac{r}{n}\right)^{nt} =$$

$$= \ln P + nt\ln\left(1+\frac{r}{n}\right),$$ where we've used a few properties of logarithms on the right side.

Now we solve for t in terms of everything else:

$$\underbrace{\ln A - \ln P}_{=\ln\left(\frac{A}{P}\right)} = nt\ln\left(1+\frac{r}{n}\right)$$

$$\boxed{\frac{\ln\left(\frac{A}{P}\right)}{n\ln\left(1+\frac{r}{n}\right)} = t}$$

6. $\log_b y$ is only defined when $y > 0$. So $\log_3(4x-7)$ is only defined when $4x-7>0$, so when $\boxed{x > \frac{7}{4}}$.

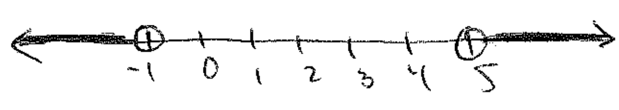
7. This is similar to 6. $\ln(x^2-4x-5)$ is only defined if $x^2-4x-5>0$, so when $(x-5)(x+1)>0$.

We want $x-5>0$ and $x+1>0$, or, $x-5<0$ and $x+1<0$.

So $x>5$ and $x>-1$, or, $x<5$ and $x<-1$,

which is $\boxed{x>5 \quad \text{or} \quad x<-1}$

On the number line this is:



8. The only time $\ln y$ is undefined is if $y \leq 0$. But $x^2 \geq 0$ for every x (square of a real number is never negative). So $x^2+1 \geq 1$ for every x.

Thus $\ln(x^2+1)$ is defined for every x. The smallest value x^2+1 takes is 1 (for $x=0$), and $\ln 1 = 0$.

Then x^2+1 can take any positive real value greater than 1 as we allow x to be any value. When x^2+1 increases, so does $\ln(x^2+1)$ (e must be raised to a higher power to get larger numbers). Since $\ln 1 = 0$ and this is the smallest output, then $\ln(x^2+1)$ can give any non-negative real value. In set notation, the set of all outputs is $\boxed{\{y \mid y \geq 0\}}$.

9. For the square root to be defined, we need a non-negative number, so we need $\log_2(x^2-3x-9) \geq 0$.

We know $\log_2 1 = 0$, so to get positive values of $\log_2 y$, we need $y \geq 1$. So we need $x^2-3x-9 \geq 1$.

This is $x^2-3x-10 \geq 0$, or $(x-5)(x+2) \geq 0$.

So $x-5 \geq 0$ and $x+2 \geq 0$, or, $x-5 \leq 0$ and $x+2 \leq 0$.

So $x \geq 5$ and $x \geq -2$, or $x \leq 5$ and $x \leq -2$,

which is $\boxed{x \geq 5 \text{ or } x \leq -2}$