Math 103 HW #2

A1. The slope of the line through the points $(-1,3)$ and $(2,4)$ is $m = \frac{3-4}{-1-2} = \frac{-1}{-3} = \frac{1}{3}$. The equation of this line is thus in the form $y = \frac{1}{3}x + b$, and we now need the y-intercept b. We can find this by plugging in either the point $x=2$, $y=4$, or $x=-1$, $y=3$, since these must satisfy the equation. Choosing the first point $(-1,3)$, we have $3 = \frac{+1}{3}(-1) + b$, so $3 = -\frac{1}{3} + b$, and $b = 3 + \frac{1}{3} = \frac{10}{3}$. (Check that using $(2,4)$ instead gives the same b). So our equation is

$$\boxed{y = \frac{1}{3}x + \frac{10}{3}}.$$

A2. Since the slope is $\frac{1}{2}$, the equation of this line is of the form $y = \frac{1}{2}x + b$. We can use the given point, $x=5$, $y=-4$, to find b: $-4 = \frac{1}{2}(5) + b$, so $-4 = \frac{5}{2} + b$, and $b = -4 - \frac{5}{2} = \frac{-13}{2}$. So our equation is $\boxed{y = \frac{1}{2}x - \frac{13}{2}}$.

A3. The given line $y = 2x + 279$ has slope $m = 2$. Since the line we want is parallel to this, it has the same slope, and so has equation of the form $y = 2x + b$. We use the given point $x = 1$, $y = 1$, to find the y-intercept: $1 = 2(1) + b$, so $b = -1$. Our equation is thus $\boxed{y = 2x - 1}$.

A4. We are given that the y-intercept of our line is $b = 5$, so the equation is of the form $y = mx + 5$. We are also given that the line is perpendicular to the line with equation $5x - 3y = 4$, which can be re-written as $-3y = -5x + 4$, which is also $y = \frac{5}{3}x - \frac{4}{3}$. This line has slope $\frac{5}{3}$, and so any line perpendicular to it has slope $\frac{-3}{5}$. The line we want has this slope, and so has equation

$$\boxed{y = \frac{-3}{5}x + 5}.$$

A5. The line given has equation $2x+y=3$, which is also $y=-2x+3$, and so this line has slope -2. The line we want is perpendicular to this, and so has slope $\frac{1}{2}$ (since $-\frac{1}{-2}=\frac{1}{2}$). The equation of our line is of the form $y=\frac{1}{2}x+b$, and we need to find the y-intercept b. We are given the x-intercept (where the line hits the x-axis) is 4, meaning it goes through the point $(4,0)$, or $x=4$, $y=0$. Using this point to find b, we have $0=\frac{1}{2}(4)+b$, or $0=2+b$, so $b=-2$. Now our line has equation $\boxed{y=\frac{1}{2}x-2}$.

A6. We multiply these polynomials as follows:

(a) $(x-3)(x-2)=x^2-3x-2x+6=\boxed{x^2-5x+6}$

(b) $(x+4)(x-5)=x^2+4x-5x-20=\boxed{x^2-x-20}$

(c) $(x+a)(x+b)=x^2+ax+bx+ab=\boxed{x^2+(a+b)x+ab}$

(d) $(3x-2)(4x+1)=12x^2-8x+3x-2=\boxed{12x^2-5x-2}$

(e) $(2x^2-1)(x+1)=\boxed{2x^3+2x^2-x-1}$

A6. (f) We could re-order the factors here to make things easier, noting $(x-2)(x+2) = x^2-4$:

$$(x-2)(x+3)(x+2) = (x-2)(x+2)(x+3) = (x^2-4)(x+3)$$

$$= \boxed{x^3+3x^2-4x-12}$$

A7. $x^2-6x+9=0$, so $(x-3)^2=0$, so $x-3=0$, and $\boxed{x=3}$ is the only solution.

A8. $x^2-7x=-12$, or $x^2-7x+12=0$, so

$(x-3)(x-4)=0$, so $x-3=0$ or $x-4=0$, so $\boxed{x=3 \text{ or } x=4}$ are the solutions.

A9. $2x^2+5x-3=0$, so $(2x-1)(x+3)=0$, so $2x-1=0$ or $x+3=0$, so $\boxed{x=\frac{1}{2} \text{ or } x=-3}$.

A10. $x^4-3x^2+2=0$. You might recognize this right away as factorable: $x^4-3x^2+2=(x^2-2)(x^2-1)$. The hint is meant to prompt this if you do not see it. If $z=x^2$, then $z^2=x^4$, so we can write $x^4-3x^2+2=z^2-3z+2$, which is a quadratic we can factor.

A10 (cont'd) Then $z^2-3z+2=(z-2)(z-1)$, and going back to $z=x^2$, we have

$$x^4-3x^2+2=z^2-3z+2=(z-2)(z-1)=(x^2-2)(x^2-1).$$

Note also that $x^2-1=(x-1)(x+1)$. So

$$(x^2-2)(x^2-1)=(x^2-2)(x-1)(x+1)=0,$$ so

$x^2-2=0$ or $x-1=0$ or $x+1=0$.

$x^2-2=0$ means $x^2=2$, so $x=\sqrt{2}$ or $-\sqrt{2}$. Now our solutions are $\boxed{x=\sqrt{2}, -\sqrt{2}, 1, \text{or } -1}$.

B1. The general method for completing the square for the equation $x^2+bx+c=0$ looks like the following: $x^2+bx+c=x^2+bx+\frac{b^2}{4}-\frac{b^2}{4}+c=$ $=\left(x+\frac{b}{2}\right)^2-\frac{b^2}{4}+c=0$, and proceed from there.

(a) $x^2+4x-6=x^2+4x+4-4-6=(x+2)^2-10=0$

So $(x+2)^2=10$, so $x+2=\pm\sqrt{10}$, so $x=-2\pm\sqrt{10}$, so the solutions are

$$\boxed{x=-2+\sqrt{10} \text{ or } -2-\sqrt{10}}$$

B1. (b) $x^2 - x - 1 = x^2 - x + \frac{1}{4} - \frac{1}{4} - 1$

$$= \left(x - \frac{1}{2}\right)^2 - \frac{5}{4} = 0.$$

So $\left(x - \frac{1}{2}\right)^2 = \frac{5}{4}$, so $x - \frac{1}{2} = \pm\sqrt{\frac{5}{4}} = \frac{\pm\sqrt{5}}{2}$, so

$x = \frac{1}{2} \pm \frac{\sqrt{5}}{2} = \frac{1 \pm \sqrt{5}}{2}$. The solutions are:

$$\boxed{x = \frac{1+\sqrt{5}}{2} \text{ or } \frac{1-\sqrt{5}}{2}}.$$

B2. $x^2 + x - 4 = 0$, so $a = 1$, $b = 1$, $c = -4$. Then

$b^2 - 4ac = 1 - 4(-4) = 17 \geq 0$, so there are solutions.

The solutions are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{17}}{2}$,

so $\boxed{x = \frac{-1+\sqrt{17}}{2} \text{ or } \frac{-1-\sqrt{17}}{2}}$

B3. $x^2 + 5x + 3 = 0$, so $a = 1$, $b = 5$, $c = 3$. Then

$b^2 - 4ac = 25 - 4(3) = 13 \geq 0$, so there are solutions.

These are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{13}}{2}$,

so $\boxed{x = \frac{-5+\sqrt{13}}{2} \text{ or } \frac{-5-\sqrt{13}}{2}}$

B4: $x^2+3x+5=0$, so $a=1$, $b=3$, $c=5$, and $b^2-4ac = 9-4(5) = -11 < 0$. Since $\sqrt{b^2-4ac} = \sqrt{-11}$ is not a real number, then there are $\boxed{\text{no real solutions}}$.

B5: $2x^2-8x+5=0$, so $a=2$, $b=-8$, $c=5$. Then $b^2-4ac = 64-4(2)(5) = 24$. The solutions are $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{8 \pm \sqrt{24}}{4} =$

$$= \frac{8 \pm 2\sqrt{6}}{4} = \frac{4 \pm \sqrt{6}}{2} \quad \left(= 2 \pm \frac{\sqrt{6}}{2}\text{, either is fine}\right).$$

So the solutions are $\boxed{x = \dfrac{4+\sqrt{6}}{2}\left(=2+\dfrac{\sqrt{6}}{2}\right) \text{ or } \dfrac{4-\sqrt{6}}{2}\left(=2-\dfrac{\sqrt{6}}{2}\right)}$

B6: $x^2-8x+15>0$, so $(x-5)(x-3)>0$.

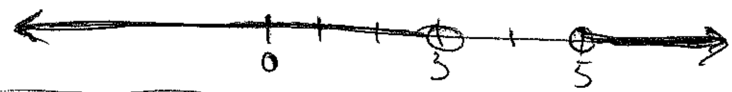
For the product to be positive, either both factors are positive or both factors are negative.

So: Either $x-5>0$ and $x-3>0$

OR $x-5<0$ and $x-3<0$.

So, $x>5$ and $x>3$, OR $x<5$ and $x<3$.

B6: (cont'd) If $x>5$ <u>and</u> $x>3$, then $x>5$. <u>If</u> $x<5$ <u>and</u> $x<3$, then $x<3$. Now the x which satisfy the inequality are all x such that $x>5$ <u>or</u> $x<3$. On the number line, this looks like:



Please pay attention to the difference between the "or" and the "and" in the solution. These are <u>key</u> in understanding what is going on.

When we say "$x>5$ and $x>3$", we are looking for values of x which satisfy <u>both</u> $x>5$ <u>and</u> $x>3$ <u>at the same time</u>. So $x=4$ <u>does not</u> satisfy this. The only x's satisfying <u>both</u> are those x such that $x>5$.

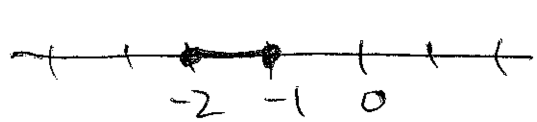
When we say $x>5$ <u>or</u> $x<3$, then we mean those x that satisfy <u>either</u> of these (There are no x which satisfy $x>5$ <u>and</u> $x<3$, but plenty which satisfy $x>5$ <u>or</u> $x<3$).

B7. $x^2+3x \le -2$, so $x^2+3x+2 \le 0$, so $(x+2)(x+1) \le 0$. The only way a product ~~less~~ of two numbers is negative (or 0) is if one is negative (or 0) and the other is positive (or 0). [We say (or 0) because of the less than or equal to $\le$]. So, either

$x+2 \le 0$ and $x+1 \ge 0$, or, $x+2 \ge 0$ and $x+1 \le 0$,

so $x \le -2$ and $x \ge -1$, or, $x \ge -2$ and $x \le -1$.

In the first case, we cannot have $x \le -2$ and $x \ge -1$ simultaneously (since $-2 < -1$, so if $x \le -2$, then $x < -1$), so this scenario is impossible. This leaves $x \ge -2$ and $x \le -1$, which can be written $-2 \le x \le -1$. So the values of x satisfying the inequality are $\boxed{\text{all } x \text{ such that } -2 \le x \le -1}$. On the number line this is:



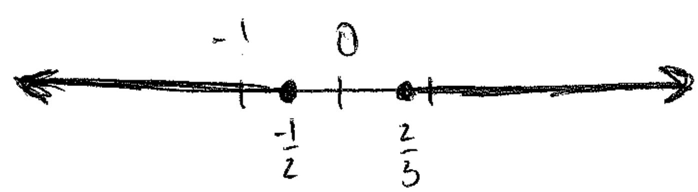
B8. $(2x+1)(3x-2) \geq 0$. Since the product of the two factors is positive (or 0), then either both products are positive (or 0) or both are negative (or 0). That is, either

$2x+1 \geq 0$ and $3x-2 \geq 0$, or, $2x+1 \leq 0$ and $3x-2 \leq 0$,

so $2x \geq -1$ and $3x \geq 2$, or, $2x \leq -1$ and $3x \leq 2$,

so $x \geq \frac{-1}{2}$ and $x \geq \frac{2}{3}$, or, $x \leq \frac{-1}{2}$ and $x \leq \frac{2}{3}$.

Now, $x \geq \frac{-1}{2}$ and $x \geq \frac{2}{3}$ means $x \geq \frac{2}{3}$, while $x \leq \frac{-1}{2}$ and $x \leq \frac{2}{3}$ means $x \leq \frac{-1}{2}$. So our final solution is the values of $\boxed{x \text{ such that } x \geq \frac{2}{3} \text{ or } x \leq \frac{-1}{2}}$, which on the number line looks like:

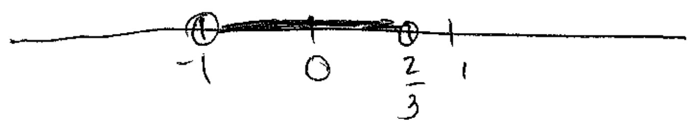


B9: $3x^2 + x - 2 < 0$, so $(3x-2)(x+1) < 0$.

For this product to be negative, one factor must be negative while the other is positive. So, either $3x - 2 > 0$ and $x + 1 < 0$, <u>or</u>, $3x - 2 < 0$ and $x + 1 > 0$,

so $x > \frac{2}{3}$ and $x < -1$, <u>or</u>, $x < \frac{2}{3}$ and $x > -1$.

But $x > \frac{2}{3}$ <u>and</u> $x < -1$ is impossible, so the only scenario to consider is $x < \frac{2}{3}$ and $x > -1$, which can be written as $\boxed{-1 < x < \frac{2}{3}}$. On the number line, this is:



B10: $x^2 - 8x + 16 \le 0$, so $(x-4)^2 \le 0$.

But the square of any real number is <u>never negative</u>, so $(x-4)^2 < 0$ is impossible for any x.

So $(x-4)^2 \le 0$ can only hold if $(x-4)^2 = 0$, which only holds for $\boxed{x = 4}$.