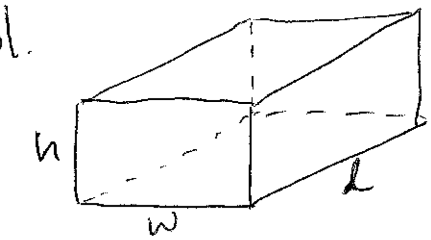
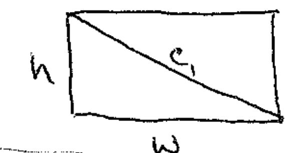
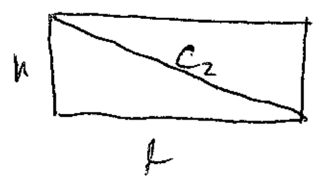
B1. 

(a) Opposite faces of the box are identical, so we just need to find the face diagonal for three different faces. For each we use the Pythagorean Theorem. For the h-by-w face: 

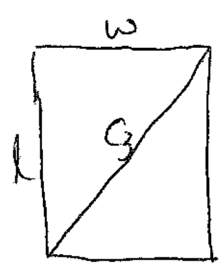
We have $h^2+w^2 = c_1^2$, so $\boxed{c_1 = \sqrt{h^2+w^2}}$

<u>Note</u>: This is as far as we can simplify.

In particular, $\sqrt{h^2+w^2}$ is <u>NOT EQUAL</u> to $h+w$.

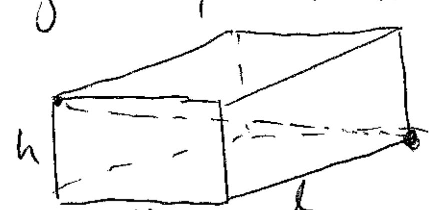
For the h-by-l face: 

$h^2+l^2 = c_2^2$, and $\boxed{c_2 = \sqrt{h^2+l^2}}$

For the l-by-w face (top or bottom): 

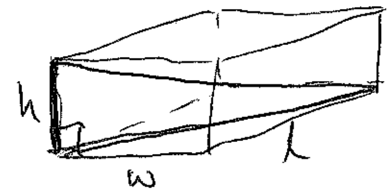
$w^2+l^2 = c_3^2$, so $c_3 = \boxed{\sqrt{w^2+l^2}}$

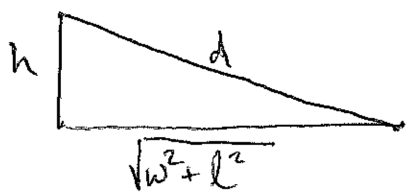
So if c_1, c_2, c_3 are the three face diagonals, we have $c_1 = \sqrt{h^2+w^2}$, $c_2 = \sqrt{h^2+l^2}$, $c_3 = \sqrt{w^2+l^2}$

(b). There are 4 space diagonals, but they are all the same length 

Consider the one pictured,

B1. (b) (cont'd) From the front top left corner to the back bottom right corner. This space diagonal is the hypotenuse of a right triangle with height h and base the face diagonal of the bottom w-by-l face:





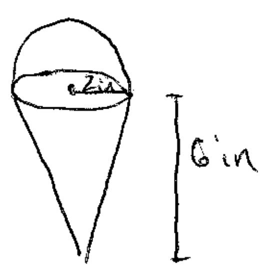
From (a), the face diagonal of the bottom face is $\sqrt{w^2+l^2}$.

Now by the Pythagorean Theorem,

$$h^2 + \left(\sqrt{w^2+l^2}\right)^2 = d^2, \quad \text{so} \quad h^2 + w^2 + l^2 = d^2.$$

(You will get the same result for each space diagonal, and its OK if you draw a different right triangle with the space diagonal as its hypotenuse).

B2.



To find the volume of all of the ice cream, we need the volume of the cone $\left(\frac{1}{3}\pi r^2 h\right)$ plus the volume of half of a sphere $\left(\frac{1}{2}\cdot\frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3\right)$

The radius of the cone and hemisphere are 2 in.

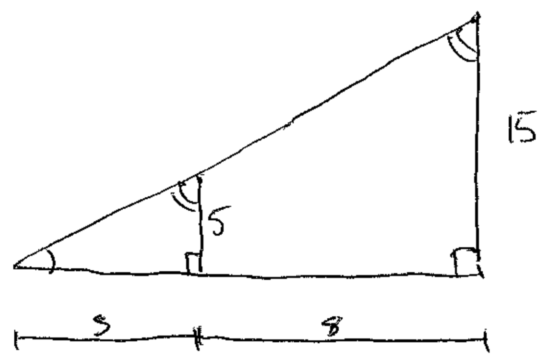
The volume of the cone is $\frac{1}{3}\pi 2^2 \cdot 6 = 8\pi \text{ in}^3$

B2. (cont'd) For the hemisphere, the volume is

$\frac{2}{3}\pi 2^3 = \frac{16}{3}\pi$ in^3. The total volume is then

$$8\pi + \frac{16}{3}\pi = \frac{24}{3}\pi + \frac{16}{3}\pi = \boxed{\frac{40}{3}\pi \text{ in}^3}$$

B3.



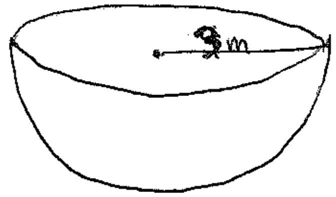
The light beam casting the shadow forms the right triangle pictured.

The larger right triangle ~~ww~~ with height 15 ft is similar to the smaller triangle of height 5 ft. The base of the smaller triangle is the shadow, which we've called s. Then the base of the larger triangle is s+8 (ft). Since this triangles are similar, we have

$$\frac{15}{5} = \frac{s+8}{s}, \text{ so } 3 = \frac{s+8}{s}, \text{ so } 3s = s+8.$$

$2s = 8 \rightarrow s = 4$. So the shadow is $\boxed{4 \text{ ft}}$ long.

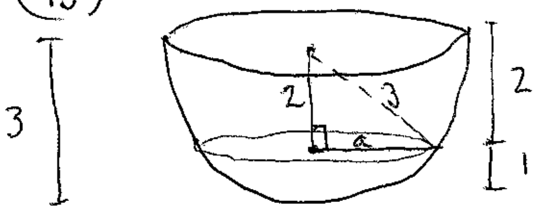
B4.



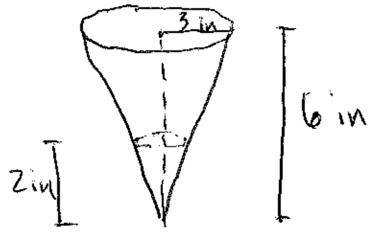
The pool is half of a sphere with radius $r = 3\text{m}$, as pictured.

(a) The surface area is the area of the cover, which is a circle of radius 3 (area πr^2), plus half of the surface area of a sphere with the same radius ($\frac{1}{2} \cdot 4\pi r^2 = 2\pi r^2$). So the whole surface area is $\pi r^2 + 2\pi r^2 = 3\pi r^2$ with $r = 3\,m$, which gives $3\pi\, 3^2 = \boxed{27\pi\ m^2}$ (don't forget units!)

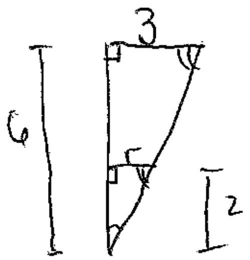
(b)



Note that the total depth of the pool is also the radius, so is 3m. If the water depth is 1 m, then the distance from the water to the top is $3 - 1 = 2\,m$, as pictured. We want the radius of the surface of the water which we've labeled a. The key observation is that the distance from the center of the top of the pool to the edge of the water is also a radius, so is 3 m. We can now use the Pythagorean Theorem, where $2^2 + a^2 = 3^2$, so $a^2 = 5$, and $\boxed{a = \sqrt{5}\ m}$

B5. 

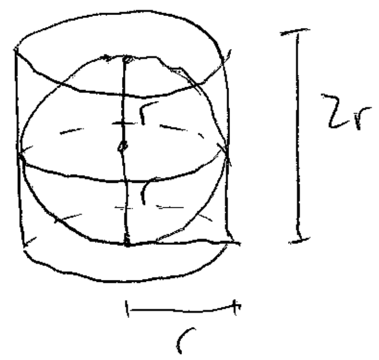
The cup is a cone as pictured, and the water in the cup <u>also</u> forms a cone, with height $h = 2\text{in}$. The volume is $\frac{1}{3}\pi r^2 h$, so we need the radius of the surface of the water. Taking a vertical cross-section of the cone, and looking at one half of it, we have the following:



We have the large triangle, with base 3 and height 6, is similar to the small triangle, with height 2 and base r.

By similar triangles, $\frac{6}{2} = \frac{3}{r}$, so $6r = 6$, so $r = 1\text{ in}$. Now the volume of the water is $\cancel{\pi r^2 h = \pi 2^2 \cdot 1 = \boxed{4\pi \text{ in}^3}}$ ← (sorry!)

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 1^2 \cdot 2 = \boxed{\frac{2\pi}{3}\text{ in}^3}$$

B6. 

Since the sphere has radius r and is inscribed in the cylinder, the cylinder has radius r and height 2r, as pictured.

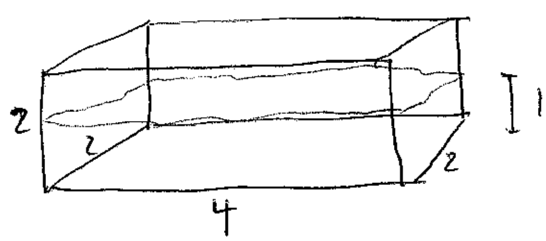
B6. (cont'd) (a) The height of the cylinder is then $h=2r$. The surface area is

$$2\pi r^2+2\pi rh = 2\pi r^2 + 2\pi r(2r) = 2\pi r^2 + 4\pi r^2$$
$$= \boxed{6\pi r^2}$$

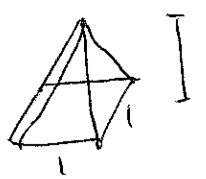
(b) The surface area of the sphere is $4\pi r^2$. The ratio of surface areas is thus

$$\frac{6\pi r^2}{4\pi r^2} = \frac{6}{4} = \boxed{\frac{3}{2} \text{ or } 3:2}$$

B7.



The aquarium has base 4 ft × 2 ft, height 2 ft, and the water is filled to 1 ft, as pictured. The pyramid

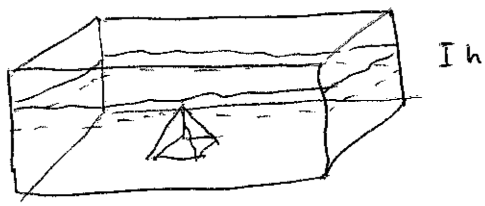


has square base 1 ft × 1 ft, and height 1 ft, so has volume

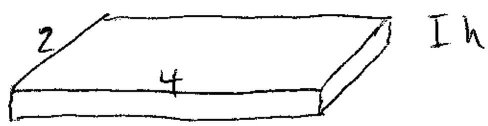
$$\frac{1}{3}Bh = \frac{1}{3}(1^2)1 = \frac{1}{3}\text{ ft}^3.$$

When the pyramid is put into the water, the water rises by some height h.

B7. (cont'd)



The key observation here is that the slice of water that has risen by a height of h must have the same volume as the submerged pyramid, which we found to be $\frac{1}{3}$ ft^3. This slice of water is the displaced volume.



This slice is a short box of height h and base the same as the aquarium, 2 ft by 4 ft. So the volume of the displaced water is $2 \cdot 4 \cdot h =$ $= 8h$ ft^3. Since this must be the volume of the pyramid, we have $8h = \frac{1}{3}$, so $h = \frac{1}{24}$ ft. We have left the units in feet, but the answer is wanted in inches. Since 12 in = 1 ft, then $\frac{1}{12}$ ft = 1 in, so $h = \frac{1}{24}$ ft $= \boxed{\frac{1}{2} \text{ in.}}$