Mathematical tricks

A selection of problems from Richard Stanley's seminar (MIT)

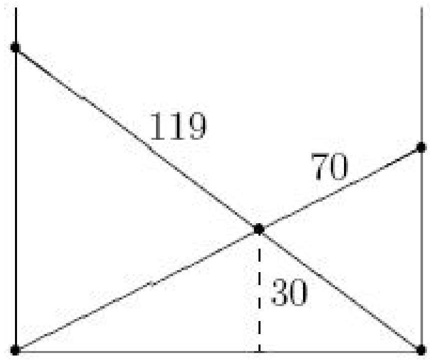
1. (MIT homework 9) (1.5 star) Find the missing term:

$$10; 11; 12; 13; 14; 15; 16; 17; 20; 22; 24; 31; 100; ?; 10000$$

2. (MIT homework 9) (1.5 star) Explain the rule which generates the following sequence: $2; 3; 10; 12; 13; 20; 21; 22; 23; 24; 25; 26; 27; 28; 29; 30; 31; 32; 33; 34; 35; 36; 37; 38; 39; 200; 201; 202; \cdots$ (Hint: Don't think mathematically!)

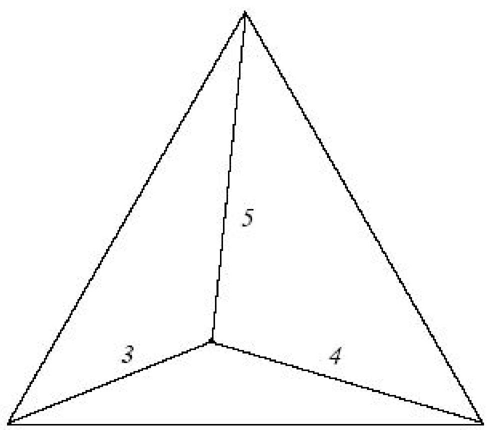
3. (MIT homework 9) (1.5 star) Why does a mirror reverse left and right but not up and down? (This is not a frivolous question.)

4. (MIT homework 9) (2.5 star) Two ladders of length 119 feet and 70 feet lean between two vertical walls so that they cross 30 feet above the ground. How far apart are the walls?



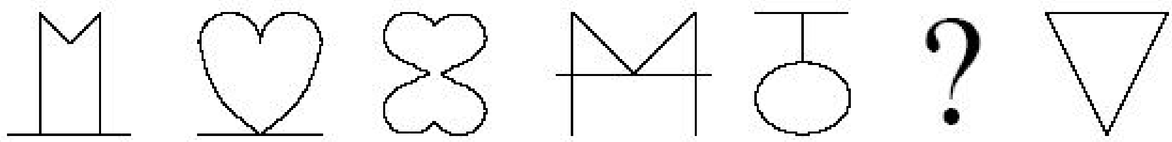
5. (MIT homework 7) (1 star) A gold chain contains 23 links. What is the least number of links which need to be cut so a jeweler can sell any number of links from 1 to 23, inclusive? Generalize.

6. (MIT homework 7) (2.5 star) A point p in the interior of an equilateral triangle T is at a distance of 3, 4, and 5 units from the three vertices of T. What is the length of a side of T?



7. (MIT homework 7) (3 star) Into how few pieces can an equilateral triangle be cut and reassembled to form a square?

8. (MIT homework 6) (1 star) Find the missing term:



9. (MIT homework 6) (1 star) (a) A drugstore received a shipment of ten bottles of a certain drug. Each bottle contains one thousand pills. The drugstore received a telegram from the drug company saying that the pills in one bottle each weigh 10 milligrams too much and should be returned immediately. How can the faulty bottle be found with only one weighing? (b) The next time the druggist received a shipment of ten bottles of the same drug, he again received a telegram from the drug company, this time saying that any any number of the bottles might contain pills each of which was 10 milligrams too heavy. Can all the faulty bottles still be determined with only one weighing?

10. (MIT homework 6) (2 star) A man and a fly both start out at the point $x = 0$ at time $t = 0$. The man walks 4 mi/hr in the positive x-direction. The fly flies at a rate of 10 mi/hr. It continually flies between the man and the point x = 0. Where will the fly be after one hour? (Do not confuse this problem with a similar one where a fly flies between two trains moving toward each other.)

11. (MIT homework 2) (2 star) (due to Oswald Jacoby; broadcast on National Public Radio, September 8, 1991) A hat contains a certain number N of blue balls and red balls. Five balls are picked randomly out of the hat (without replacement). The probability is exactly 1/2 that all five balls are blue. What is the smallest possible value of N? Try to give a simple argument avoiding factorials, binomial coefficients, etc.

12. (MIT homework 1) (1 star) In this problem, "knights" always tell the truth and "knaves" always lie. In (a)-(c), all persons are either knights or knaves.

    (a) There are two persons, A and B. A says, "At least one of us is a knave." What are A and B?

    (b) A says, "Either I am a knave or B is a knight." What are A and B?

    (c) Now we have three persons, A, B, and C. A says, "All of us are knaves." B says, "Exactly one of us is a knight." What are A, B, and C?

    (d) Now we have a third type of person, called "normal,"who sometimes lies and sometimes tells the truth. A says, "I am normal." B says, "That is true." C says, "I am not normal." Exactly one of A, B, C is a knight, one is a knave, and one is normal. What are A, B, and C?