Linear Programs . . .

specific

$$\begin{aligned} \text{MIN } & 40x_1 + 36x_2 \\ \text{s.t. } & x_1 \le 8 \\ & x_2 \le 10 \\ & 5x_1 + 3x_2 \ge 45 \\ & x_1, x_2 \ge 0 \end{aligned}$$

standard $\Downarrow$ form

$$\begin{aligned} \text{MIN } & 40x_1 + 36x_2 \\ \text{s.t. } & x_1 + s_1 = 8 \\ & x_2 + s_2 = 10 \\ & 5x_1 + 3x_2 - s_3 = 45 \\ & (x_1, x_2, s_1, s_2, s_3) \ge 0 \end{aligned}$$

$\rightsquigarrow$

general

$$\begin{aligned} \text{MIN } & \vec{c}^T \vec{x} \\ \text{s.t. } & A\vec{x} \le \vec{b} \\ & \vec{x} \ge \vec{0} \end{aligned}$$

standard $\Downarrow$ form

$$\begin{aligned} \text{MIN } & \vec{c}^T \vec{x} \\ \text{s.t. } & (A \,\vdots\, I)\begin{pmatrix} \vec{x} \\ \vec{s} \end{pmatrix} = \vec{b} \\ & (\vec{x}, \vec{s}) \ge \vec{0} \end{aligned}$$

Why do we need the standard form?

.... in the inspection problem

EXTREME Pts.

$(8, 10)$ $x_1 \le 8$, $x_2 \le 10$ Active

$(3, 10)$ $x_2 \le 10$, $5x_1 + 3x_2 \ge 45$ Active

$(8, 5/3)$ $x_1 \le 8$, $5x_1 + 3x_2 \ge 45$ Active

Theorem $(\vec{x}, \vec{s})$ is a basic feasible solution of a LP in standard form iff $\vec{x}$ is an extreme pt.

.... in the inspection problem

EXTREME Pts		B.F.S.
$(8, 10)$	$\longrightarrow$	$(8, 10, 0, 0, 35)$ $x_1\ x_2\ s_3$ BASIC
$(3, 10)$	$\longrightarrow$	$(3, 10, 5, 0, 0)$ $x_1\ x_2\ s_1$ BASIC
$(8, 5/3)$	$\longrightarrow$	$(8, 5/3, 0, \frac{25}{3}, 0)$ $x_1\ x_2\ s_2$ BASIC

SOLVE

$$(A \,\vdots\, I)\begin{pmatrix} x \\ s \end{pmatrix} = b$$

There are n free variables in this system (nonbasic $=0$).