Hmk #4 Key
Math 323
Fall 2009

5-25 (a) $B = \begin{pmatrix} 3 & 1 \\ 0 & -1 \end{pmatrix}$. $B^{-1} = \begin{pmatrix} 1/3 & 1/3 \\ 0 & -1 \end{pmatrix}$. (max version)

(b) $V = C_B^T B^{-1} = (0,1)\begin{pmatrix} 1/3 & 1/3 \\ 0 & -1 \end{pmatrix} = (0,-1)$

(c) $\bar{c}_1 = c_1 - V a^{(1)} = 8 - (0,-1)\begin{pmatrix} 1 \\ -4 \end{pmatrix} = 8 - 4 = 4 > 0 \Rightarrow$ improving

$\bar{c}_2 = c_2 - V a^{(2)} = -5 - (0,-1)\begin{pmatrix} 2 \\ 1 \end{pmatrix} = -5 + 1 = -4 < 0$ NOT improving

(d) $\Delta x_B = -B^{-1} a^{(1)} = -\begin{bmatrix} 1/3 & 1/3 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -4 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$. So, $\Delta x^{(1)} = (1,0,-3,-4)$

$x_B^{(0)} = B^{-1} b = \begin{pmatrix} 1/3 & 1/3 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 7 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. So $x^{(0)} = (0,0,2,1)$

$x^{(0)} + \lambda \Delta x^{(1)} = (0,0,2,1) + \lambda(1,0,-3,-4)$. If $\lambda = 1/4$ then $x_4 = 0$

Thus x_4 leaves the basis.

$B^{-1}_{new} = E \cdot B^{-1}_{old}$ where $E = \begin{bmatrix} 1 & -3/4 \\ 0 & 1/4 \end{bmatrix}$ (x_3, x_4). So $B^{-1}_{new} = \frac{1}{12}\begin{bmatrix} 4 & 1 \\ 0 & -3 \end{bmatrix}$.

7-5 (b) tighten, increases, rate increases (steeper)

(d) relax, increases, rate decreases (flatter)

7-6 (b) tighten, decrease

(d) tighten, increase

7-7 (b) decrease, less steep

(d) decrease, more steep

7-11 (a) V_1 ($/ticket) dollar change in profit per ticket increase > 10,000 tickets

V_2 ($/ticket) dollar change in profit per ticket allocation to x_3 above $\frac{1}{2}x_2$

V_3 ($/ticket) dollar change in profit per ticket increase > 500 tickets.

(b) $V_1 \geq 0$ The marginal price/ticket is non-negative if 10,000 increases

$V_2, V_3 \leq 0$ marginal price/ticket decrease the optimal profit if 500 increases or allocation of tickets to $x_3 > \frac{1}{2}x_2$.

(c) $V_1 + V_3 \geq 0$

$V_1 + V_2 \geq 45$

$V_1 - \frac{1}{2}V_2 \geq 100$

(d) min $10,000 V_1 + 500 V_3$

(e) $45(3166\frac{2}{3}) + 100(6333\frac{1}{3}) = 775,833\frac{1}{3}$ (B)

$10,000(8\frac{2}{3}) + 500(-8\frac{2}{3}) = 775,833\frac{1}{3}$ (D)

(f) $x_1 + x_2 + x_3 = 10,000$ or $V_1 = 0$ or both

$x_2 - \frac{1}{2}x_3 = 0$ or $V_2 = 0$ or both. $x_1 = 500$ or $V_3 = 0$ or both.

(g) $x_1 = 0$ or $V_1 + V_3 = 0$ or both. $x_2 = 0$ or $V_1 + V_2 = 45$ or both.

$x_3 = 0$ or $V_1 - \frac{1}{2}V_2 = 100$ or both.

7-11 (h) $500 + 3166^{2/3} + 6333^{1/3} \geq 10{,}000$ [or] $v_1 = 81^{2/3} \neq 0$

$3166^{2/3} - \frac{1}{2}(6333^{1/3}) \geq 0$ [or] $v_2 = -36^{2/3} \neq 0$

$500 \geq 500$ [or] $v_3 = -81^{2/3} \neq 0$

$x_1 = 500 \neq 0$ [or] $81^{2/3} - 81^{2/3} \geq 0$

$x_2 = 3166^{2/3} \neq 0$ [or] $81^{2/3} - 36^{2/3} \geq 45$

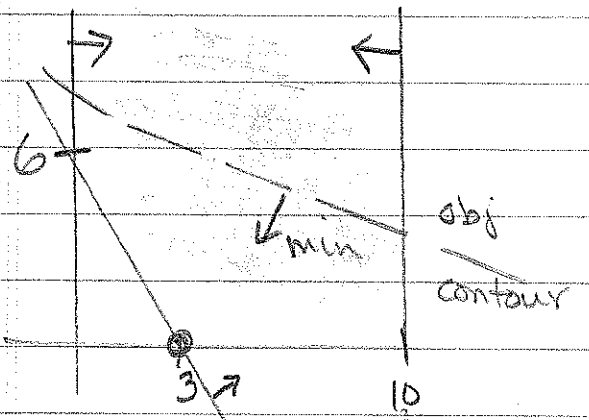
$x_3 = 6333^{1/3} \neq 0$ [or] $81^{2/3} - \frac{1}{2}(-36^{2/3}) \geq 100$

7-14(b) $\min 4x_1 + 10x_2$

s.t. $2x_1 + x_2 \geq 6$

$x_1 \leq 10$

$x_1, x_2 \geq 0$



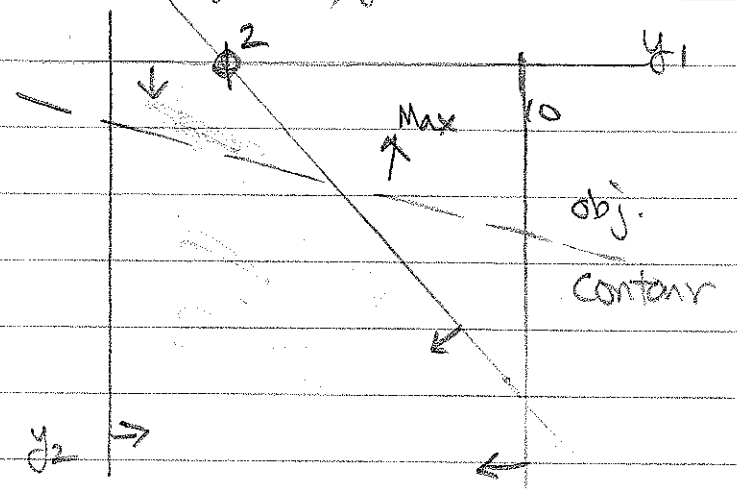
$x^* = (3,0)$ $f(x^*) = 12$

$\max 6y_1 + 10y_2$

s.t. $2y_1 + y_2 \leq 4$

$y_1 \leq 10$

$y_1 \geq 0, y_2 \leq 0$



$y^* = (2,0)$ $g(y^*) = 12$

7-15(b) $\{x_1, x_3\} = X_B$. So, $B = \begin{bmatrix} 2 & 5 \\ 0 & 3 \end{bmatrix}$ and $B^{-1} = \frac{1}{6}\begin{bmatrix} 3 & -5 \\ 0 & 2 \end{bmatrix}$

Thus, $v = C_B^T B^{-1} = (6,21)\begin{bmatrix} 3 & -5 \\ 0 & 2 \end{bmatrix}\frac{1}{6} = (3,2)$

7-12(b) $\min 20v_1 + 25v_3$

s.t. $v_1 + v_2 + 9v_3 \geq 44$

$v_1 - v_2 - 3v_3 \geq -3$

$v_1 + v_3 \geq 15$

$-v_1 - v_3 \geq -56$

v_1 free, $v_2 \geq 0$, $v_3 \leq 0$

5-25 8 pts	7-11 16 pts	
7-4a 2 pts	7-12b 3 pts	
7-5bd 4 pts	7-14b 6 pts	
7-6 bd 4 pts	7-15b 2 pts	
7-7bd 4 pts	49 pts	