Key for homework 4

Problem 10 (section 3.1). Can you walk through each doorway once and only once?
In other words, is the multi-graph traversable. The degree sequence for the constructed multi-graph is $2, 2, 2, 3, 4, 5, 8$. (Note that the room with no doors has no edges (doors) and can be excluded.) Theorem 3.3 tells us that if there are exactly two odd vertices then the multi-graph is traversable. The degree sequence meets the condition of the theorem (vertices with degrees 3 and 5). Thus, it is possible to walk through each doorway once and only once. You must also give a route that does so to get full credit.

Problem 21. (section 3.2) If the degree restriction is replaced with "deg v $\geq (p-1)/2$" then the theorem is false.
One way to show this is to use a counterexample as described in the hamiltonian sufficient condition pdf on my home page. There it states that any complete bipartite graph $K_{d,d+1}$ is a counterexample. I chose to construct a graph that would work for problems 21 and 23. G has 5 vertices. 4 of the vertices are of degree 2 and the fifth is of degree 4. Two triangles sharing a cutvertex of degree 4 (looks like a bowtie). G meets the new condition (deg v ≥ 2), but no hamiltonian cycle exists. Assume a ham cycle exists in G, then every edge incident with each degree 2 vertex must be included in the cycle. Thus, every edge of G is included, and the result is not a cycle.

Problem 23 (section 3.2). Every eulerian graph is hamiltonian.
The statement is false. The graph G described in problem 21 (bowtie graph) is eulerian (all vertices are even) but, as was explained in problem 21, it is not hamiltonian.

Problem 27 (section 3.2). Determine if the graph of figure 3.21 is hamiltonian.
The graph is not hamiltonian. To see this, assume that a ham cycle exists. Thus, every edge incident with the degree 2 vertices must be included. As a result all vertices except the degree 10 vertex in the center are part of the constucted cycle. However, adding any additional edges to reach the degree 10 vertex forces a vertex currently in the cycle to have a degree greater than 2 which is not allowed. Thus, figure 3.21 does not have a ham cycle.

Problem 30 (section 3.2). Solve the TSP problem for the network in figure 3.22.
We have no algorithm to do this, so we resort to complete enumeration. The network has 3 hamiltonian cycles. The costs of these three cycles are \$200, \$158, and \$ 157. The cheapest is the TSP solution (A to B to D to C and back to A).