Hmk #3
Math 323
Fall 2009

~~3-17~~ bd 6pts | 5-8 10 pts
3-18 bd 6pts | 5-16b 4pts
5-3f 4pts | 5-23 10pts
5-5 10pts | 5-15 8 pts

58 pts

3-17 (b) (0, 0, 0) violates constraints (2) and (3).

$$\min a_1 + a_2$$
$$\text{s.t. } 3w_1 + w_2 + 2w_3 \le 9$$
$$4w_1 + 4w_2 + a_1 \ge 6$$
$$w_1 - w_2 - w_3 - a_2 = -2$$
$$w_1, w_2, a_1, a_2 \ge 0$$

(0, 0, 0, 6, 2) is a starting solution

(d) (0, 0) violates constraint (3).

$$\min a_1$$
$$\text{s.t. } (w_1 - 5)^2 + w_2^2 \le 25$$
$$3w_1 - w_2 = 0$$
$$w_1 + a_1 \ge 2$$
$$w_2, a_1 \ge 0$$

(0, 0, 2) is a starting solution.

3-18 (b) Since $\vec{y}$ is a global opt. to Phase I and $y_4 + y_5 = 0$, we know that (22, 4, 3) is a feasible solution for the Phase II model.

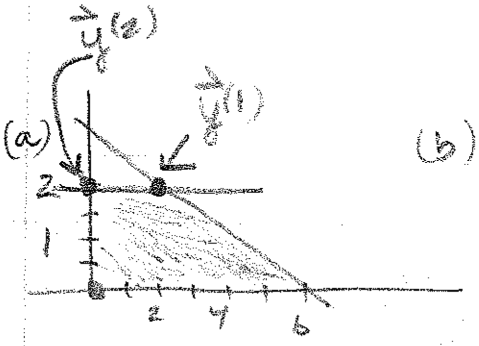
(d) Since $\vec{y}$ is a local opt. with a Phase I objective value of 6 we do not have a feasible solution for the Phase II model. Either restart from a new initial Phase I solution or conclude nothing.

5-3(f) Let $x_1 = -x_1'$ and $x_2 = x_2'' - x_2'$ then we have

$$\max -4x_1' - x_2'' + x_2'$$
$$\text{s.t. } 4x_1' - x_2'' + x_2' + 7x_3 - s_1 = 9$$
$$x_1' - x_2'' + x_2' + 3x_3 + s_2 = 14$$
$$x_1', x_2'', x_2', x_3, s_1, s_2 \ge 0$$

$$A = \begin{bmatrix} 4 & -1 & 1 & 7 & -1 & 0 \\ 1 & -1 & 1 & 3 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 9 \\ 14 \end{bmatrix}, \text{ and } \vec{c} = [4\ -1\ 1\ 0\ 0\ 0]^T$$

5.5 (a)



(b) $y_1 + 2y_2 + y_3 = 6$

$y_2 + y_4 = 2$

$y_1, y_2, y_3, y_4 \geq 0$

(c) $\{y_1, y_2\}$ basis $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ det = 1 ⇒ lin. ind.

$\{y_2, y_3\}$ basis $\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$ det = -1 ⇒ lin. ind.

$\{y_1\}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ does not span $\mathbb{R}^2$ not a basis

$\{y_2, y_4\}$ basis $\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ det = 2 ⇒ lin. ind.

$\{y_2, y_3, y_4\}$ $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ l.c. ⇒ not a basis

$\{y_1, y_3\}$ $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ det = 0 ⇒ lin. dep. not a basis.

col 1 = 2 col 2 + 1 col 3

$\begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(d) $\{y_1 y_2\}$ has basic solution $\vec{y}^{(1)} = (2, 2, 0, 0)$. Since $\vec{y} \geq 0$ ⇒ feasible

$\{y_2 y_3\}$ " " " $\vec{y}^{(2)} = (0, 2, 2, 0)$. " " " "

$\{y_2 y_4\}$ " " " $\vec{y}^{(3)} = (0, 3, 0, -1)$. Since $\vec{y} \not\geq 0$ ⇒ infeasible

(e) Every b.f.s. corresponds to an extreme pt.

5-8 (a) $(0, 0, 2, 1)$ is the b.f.s. (b) $\Delta x^{(1)} = (1\ 0\ -3\ -4)$

$\Delta x^{(2)} = (0\ 1\ -1\ \ 1)$

(c) Check $A\,\Delta x = 0$. If true then $A(x + \lambda\Delta x) = b\ \forall \lambda \geq 0$.

(d) $c^T \cdot \Delta x^{(1)} = 4$ ⇒ improving

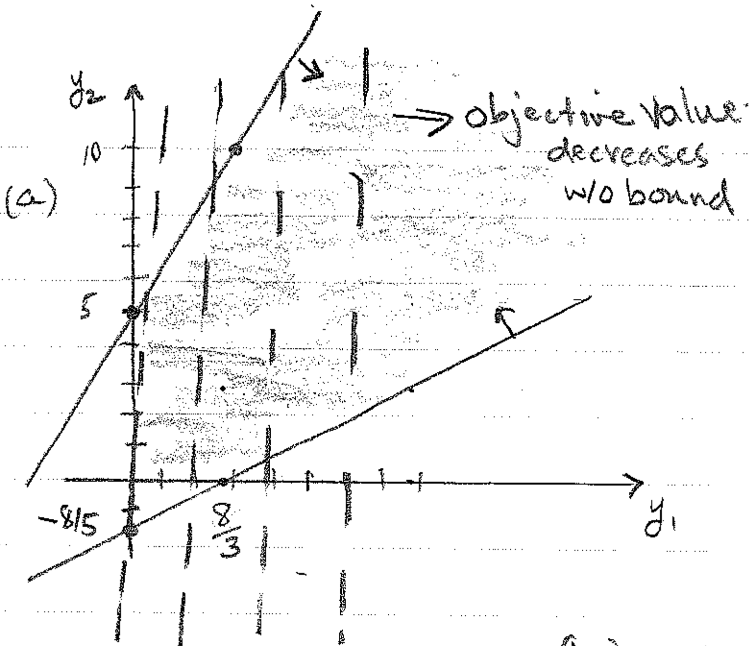
$c^T \cdot \Delta x^{(2)} = -4$ ⇒ not improving

(e) λ for $\Delta x^{(1)}$ is $1/4$

λ for $\Delta x^{(2)}$ is 2

(e) cont. The new basis if $\Delta x^{(1)}$ is used are the cols of A corresponding to x_1 and x_3 $\begin{bmatrix} 13 & 3 \\ -4 & 0 \end{bmatrix}$. For $\Delta x^{(2)}$ we have $\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$ for x_2 and x_4.

5-15 (a)



(b) $\min -10y_1 + y_2$

s.t. $-5y_1 + 3y_2 + y_3 = 15$

$3y_1 - 5y_2 + y_4 = 8$

$y_1, y_2, y_3, y_4 \geq 0$

(c)

	y_1	y_2	y_3	y_4	
$\min \vec{c}$	-10	1	0	0	$\vec{b}$
A	-5	3	1	0	15
	3	-5	0	1	8
$t=0$	N	N	B	B	
$\vec{y}^{(0)}$	0	0	15	8	$\vec{c}\cdot y^{(0)} = 0$
$\vec{\Delta y}^{(1)}$	1	0	5	-3	$\vec{c}\cdot\vec{\Delta y}^{(1)} = -10 \Leftarrow$ improving y_1 enters basis
$\vec{\Delta y}^{(2)}$	0	1	-3	5	$\vec{c}\cdot\vec{\Delta y}^{(2)} = 1$
	–	–	–	$8/3$	$\lambda = 8/3$ y_4 leaves basis
$t=1$	B	N	B	N	
$\vec{y}^{(1)}$	$8/3$	0	$28\frac{1}{3}$	0	$\vec{c}\cdot\vec{y}^{(1)} = -26\frac{2}{3}$
$\vec{\Delta y}^{(2)}$	$5/3$	1	$16/3$	0	$\vec{c}\cdot\vec{\Delta y}^{(2)} = -15\frac{2}{3} \Leftarrow$ improving y_2 enters
$\vec{\Delta y}^{(4)}$	$-1/3$	0	$-5/3$	1	$\vec{c}\cdot\Delta y^{(4)} = 3\frac{1}{3}$

$$\vec{y}^{(2)} = (8/3, 0, 28\tfrac{1}{3}, 0) + \lambda\,(5/3, 1, 16/3, 0)$$

Since $\vec{\Delta y}^{(2)}$ has no negative component there is no upper bound on $\lambda > 0$. Hence, $\vec{c}^T\vec{y}^{(2)}$ decreases w/o bound.

5-16 (b) (0,0,0,0) violates constraints (2) and (3).

$$\min\ a_1 + a_2$$

s.t.

$$w_1 - 18w_2 + s_1 = 9$$

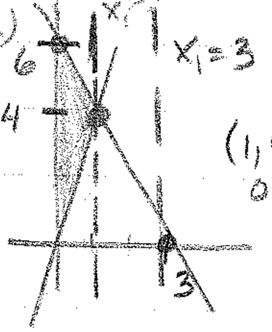
$$w_3 + w_4 + a_1 - s_2 = 14$$

$$w_1 + w_2 - 2w_3 - 3w_4 + a_2 = 1$$

$$w_1, w_2, w_3, w_4, a_1, a_2, s_1, s_2 \geq 0$$

The initial Phase I b.f.s. is (0,0,0,0,14,1,9,0) with basic variables $\{a_1, a_2, s_1\}$.

5-23 (a)



(1,4) is optimal

(b)

$$\max\ x_1$$

s.t.

$$6x_1 + 3x_2 + s_1 = 18$$

$$12x_1 - 3x_2 + s_2 = 0$$

$$x_1, x_2, s_1, s_2 \geq 0$$

(c)

	x_1	x_2	s_1	s_2	
max c	1	0	0	0	b
A	6	3	1	0	18
	12	-3	0	1	0
	N	N	B	B	
$x^{(0)}$	0	0	18	0	$c^T\cdot x^{(0)}=0$
$\Delta x^{(1)}$	1	0	-6	-12	$c^T\cdot \Delta x^{(1)}=1$ ← improving
$\Delta x^{(2)}$	0	1	-3	3	$c^T\cdot \Delta x^{(2)}=0$
	—	—	$\frac{18}{+6}$	$\frac{0}{+12}$	$\lambda=0$
	B	N	B	N	
$x^{(1)}$	0	0	18	0	$c^T x^{(1)}=0$
$\Delta x^{(2)}$	1/4	1	$-9/2$	0	$c\cdot\Delta x = 1/4$ ← improving
$\Delta x^{(4)}$	$-1/12$	0	$-1/2$	1	$c\cdot\Delta x = -1/12$
	—	—	4	—	$\lambda=4$
	B	B	N	N	
	1	4	0	0	

← one more set of Δx's to verify optimality

(d) (0,0) to (0,0) to (1,4)

(e) $\lambda = 0 \Rightarrow$ degeneracy. But we "move" from one b.f.s to another b.f.s. representing the same extreme pt.