Key for homework 2

2.2 The degree sequence $2, 3, 3, 4, 4, 5$ implies that if such a graph G exists, then it must have six vertices. We notice that three of these vertices are even and three are odd. Theorem 2.2 states that every graph contains an even number of odd vertices. Hence, a graph with three odd vertices cannot exist.

2.4 Assume there exists a graph G with degree sequence $1, 3, 3, 3$. Thus, G must have four vertices. We notice that the maximum degree of a vertex in an order four graph is three, and that three vertices in G achieve this value. Consider the construction of such a graph. Without loss of generality, let v_1 denote the degree one vertex. Each of the remaining vertices, v_2, v_3, and v_4 must have an edge incident with v_1, otherwise each vertex cannot be of degree three. This is impossible since v_1 is degree one. Hence, a graph with degree sequence $1, 3, 3, 3$ cannot exist.

2.13 The conditions of the problem statement require no self-loops, no multiple edges, noe edges between spouses, and a maximum degree of 6. Further, since each person reports a different number of handshakes (degree) the degree sequence (excluding the person reported to) must be 0,1,2,3,4,5,6. The sum of these degrees is 21. Since the sum of the degrees of a graph must be even the degree of person reported to must be odd. The following degree pairs (couples) satisfy these conditions: $(0, 6)$, $(1, 5)$, $(2, 4)$, and $(3, 3)$. (a) I shook hands with 3 people. (b) My spouse shook hands with 3 people.