Csci 688 (Kincaid/MWF 9-9:50 a.m. Jones Hall)

Scale-Free Networks

Answers to selected exam questions

1. This sequence is not a path since vertex v_3 is repeated. The sequence meets the requirement for both a walk and a trail. The sequence is most appropriately labelled a trail since any trail is a walk.

2. We know that G has exactly two components, each of which is a complete graph. We also know that a complete graph of order r is $r-1$ regular. Thus, each vertex has degree $r-1$ and the graph must have $r(r-1)/2$ edges. The first component of G is order x, and must have $x(x-1)/2$ edges. The second component is order $p-x$ and must have $(p-x)(p-x-1)/2$ edges. This verifies the formula for $f(x)$.
   Although not necessary for a correct answer, it is interesting to note that solving $f'(x) = 0$ and substituting the result ($x = p/2$) for x into $f(x)$ yields $q = (p^2 - 2p)/4$. The resulting graph when $p = 6$ is two K_3 components.

3. H1 and H2 fail. H3 and H4 are both satisfied by this graph. H3 is satisfied by noting that for $p = 6$ we need only check on vertices of degree 2 or less. There are 0 vertices of degree 1 and 1 vertex of degree 2 or less. To check H4 first reorder the degree sequence from low to high $d_1 = 2$, $d_2 = 3$, $d_3 = 3$, $d_4 = 4$, $d_5 = 4$, and $d_6 = 4$. Thus $d_k <= k$ is satisfied for k = 3,4,5, and 6. Since the minimum value for any of the possible sums is 7 the condition is met.

4. If p is even then so is $(p-2)/2 = (p/2) - 1$. The minimum degree requirement is $(p/2) - 1$. Thus, it is possible to construct a graph consisting of two complete components, each with $p/2$ vertices of degree $(p/2) - 1$.

6. (a) The average shortest hop distance is the sum of all pairwise shortest hop distances divided by the total number of hop distances possible. Since the graph is undirected the distance matrix is symmetric and the main diagonal contains all zero entries (no self-loops). The sum of the entries above (or below) the diagonal is 84. I accepted a variety of solutions for the average 168/81, 84/45, or 84/36 = 168/72. (The answer depends upon whether you account for the diagonal entries in the average or not.)
   (b) The clustering coefficient C requires us to first compute C_i for each vertex i. That is, we compute the number of triangles incident to i and divide by the number of triples centered on i. The numerator will be 1 for three vertices and zero for all other vertices. The denominators for the three non-zero numerators are 3, 3, and 6. The formula for C asks us to sum these values $1/3 + 1/3 + 1/6$ for a total of 5/6 and then average this value over the total number of vertices yielding 5/6/9 or 5/54.
   (c) The pdf is given by $p(1) = 4/9$, $p(2) = 2/9$, $p(3) = 2/9$; and $p(4) = 1/9$. The cdf is given by $P(4) = 1/9$, $P(3) = 3/9$, $P(2) = 5/9$ and $P(1) = 1$.