Math 323: Exam 1 Key—Fall 2009

Operations Research I–Deterministic Models

1. (a) True.

1. (b) False. All components of a feasible, improving search direction must be non-negative for the direction to be unbounded.

1. (c) False. Replace 7 with 11.

1. (d) False. Replace *multiple optimal* with *degenerate*.

1. (e) A variety of ways to interpret this question. One acceptable option: False, replace *improving* with *feasible*.

2. (a) $x_2 \geq 0$ and $x_4 \geq 0$ are active in addition to the main constraints ($Ax = b$).

2. (b) In $\mathcal{R}^3$ a basis requires three linearly independent columns of A. Since the solution is degenerate (a basic variable value is 0) there multiple bases associated with the extreme point. They are NBBBN, BBNBN, and NBNBB.

3. The EOQ model, due to its restrictive assumptions, sets the safety stock level to zero. We know that demand is not constant and any variability that leads to an increase in demand for a given period will lead to lost sales since there is no safety stock (similarly a decrease in demand leads to an increase in holding costs). A reasonable hedge against the overly restrictive assumptions would be to allow safety stock to be greater than zero.

4. (a) Phase I objective is always unimodal. Coupled with a convex (linear constraints) feasible region we conclude that any local optima is a global optima. Since the sum of the artificial variables is non-zero and the solution is a global optimum to the Phase I problem we conclude that the original problem is infeasible.

4. (b) We can conclude nothing. The local optimum may not be global (due to nonlinear Big-M objective). Either increase M or restart the Big-M search at a new solution.

5. The Phase I problem below has an initial basis corresponding to the columns associated with a_1, s_2 and a_2 (note identity matrix). The initial basic feasible solution is $(x_1, x_2, x_3, a_1, s_1, s_2, a_2) = (0, 0, 0, 5, 0, 4, 0)$.

$$
\begin{aligned}
Minimize\ & a_1 + a_2 \\
subject\ to\ & x_1 + x_2 + x_3 \qquad\qquad + a_2 = 0 \\
& 3x_1 - 7x_2 + x_3 + a_1 - s_1 \qquad \geq 5 \\
& x_1 + 2x_2 - 5x_3 \qquad + s_2 \leq 4 \\
& x_1, x_2, x_3, s_1, s_2, a_1, a_2 \geq 0
\end{aligned}
$$

6. (a) $min\ \Sigma_{j\in\{F,C,B\}}(320x_{1j} + 400x_{2j} + 360x_{3j} + 290x_{4j})$.

6. (b)

$$\Sigma_{j\in\{F,C,B\}}x_{1j} \le 20,\ \Sigma_{j\in\{F,C,B\}}x_{2j} \le 16,$$

$$\Sigma_{j\in\{F,C,B\}}x_{3j} \le 25,\ \text{and}\ \Sigma_{j\in\{F,C,B\}}x_{4j} \le 18$$

for cargo weight availability. While for compartment weight capacity

$$\Sigma_{i=1}^{4}x_{iF} \le 12,$$

$$\Sigma_{i=1}^{4}x_{iC} \le 18,\ \text{and}$$

$$\Sigma_{i=1}^{4}x_{iB} \le 10$$

6. (c)

$$500x_{1F} + 700x_{2F} + 600x_{3F} + 400x_{4F} \le 7000,$$

$$500x_{1C} + 700x_{2C} + 600x_{3C} + 400x_{4C} \le 9000,\ \text{and}$$

$$500x_{1B} + 700x_{2B} + 600x_{3B} + 400x_{4B} \le 5000.$$

6. (d) $\Sigma_{i=1}^{4}x_{iF}/12 = \Sigma_{i=1}^{4}x_{iC}/18 = \Sigma_{i=1}^{4}x_{iB}/10$.

7. (a) Solving $A\Delta x^{(3)} = 0$ yields $c = 1/3$ and $b = -2d/3$. Requiring $c^t\Delta x^{(3)} > 0$ results in $a > -1/2$. Thus, any choice of $b < 0$ such that $d = -3b/2$ works.

7. (b) For $\Delta x^{(3)}$ to be an unbounded, improving, feasible search direction we use the results of (a) ($c = 1/3$, $b = -2d/3$, and $a > -1/2$) to conclude that any choice of $b > 0$ such that $d = -3b/2$ works.

8. (a) Solve $Bx_B = b$ and obtain the basic feasible solution $\vec{x} = (1, 4, 0, 0, 2)$.

8. (b.) Verify that $A\Delta x^{(3)} = 0$ and $A\Delta x^{(4)} = 0$.

8. (c.) $c^t\Delta x^{(3)} = 4/3$ and $c^t\Delta x^{(4)} = -5/3$. Thus, $\Delta x^{(3)}$ is improving and $\Delta x^{(4)}$ is not.

8. (d.) $\lambda = min\{2/(2/3)\} = 3$.

8. (e.) $c^t(1, 4, 0, 0, 2) = 9$.