MATH 323 (Kincaid/Fall 1996)

Examination 1

October 9

Name: ____________________________

1. The examination is closed book except for one page of 8.5 inch by 11 inch, hand written notes.
2. You may use a calculator, but other than this and your page of notes, you should have only writing instruments at your desk. (No trumpets, trombones, etc.)
3. Write with a soft pencil or a pen (blue or black ink).
4. Write only on the exam sheets provided.
5. Watch the blackboard or overhead for corrections and/or explanations.
6. Read carefully. Problems often have several parts or subsections.
7. The point distribution for each question out of 100 percent is shown in ( ).

1. *Circle 'T' or 'F' according to whether each statement below is 'true' or 'false'. If the answer is 'F' explain what changes would make it true.* (15%)

T F (a) The set of all points (x_1, x_2) satisfying $2x_1 - 3x_2 + 4x_3 = 2$ and $x_1 + x_2 = 4$ is convex.

T F (b) Consider a minimize optimization model with objective function $f(x)$. A feasible direction is improving at $x^{(1)}$ for small positive values of the step size λ if the $\nabla f(x^{(1)})\Delta x > 0$.

T F (c) If $\Delta x = (0.1, -0.1)$ is improving and feasible at a point x, then $\Delta x = (10, -10)$ is also improving and feasible.

T F (d) The constraint ${x_1}^2 + 2x_2 \leq 6$ is active at the point $x = (-2, 1)$.

T F (e) Every local optima to the Phase I model associated with an optimization model with a non-unimodal objective function and convex constraint set is a global optima.

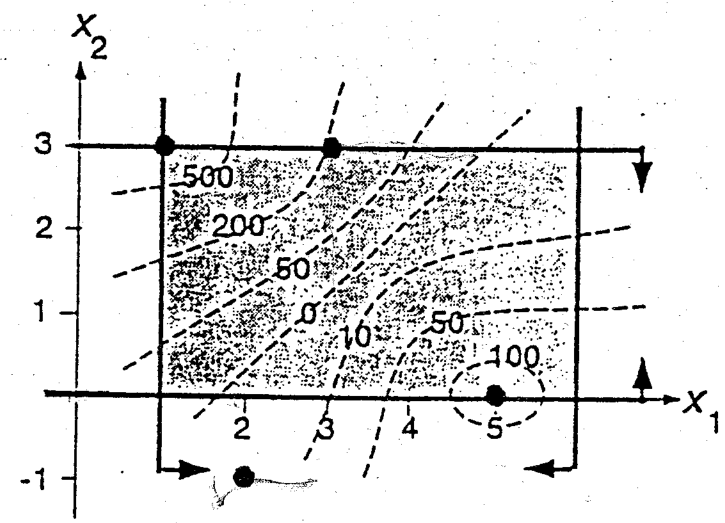
2. (6%) Prove that the set of points defined by ${x_1}^2 + {x_2}^2 \geq 9$, ${x_1}^2 + {x_2}^2 \leq 25$ $x_1 \geq 0$, and $x_2 \geq 0$ is **not** convex.

3. (15%) In the plot below, indicate whether each of the move directions at the specified points is a feasible direction and/or an improving direction. Explain how you reached your conclusion. Assume that we wish to maximize the objective function in the model.

(a) $\Delta x = (0, 1)$ at $(2, -1)$

(b) $\Delta x = (0, 1)$ at $(3, 3)$

(c) $\Delta x = (-10, 0)$ at $(3, 3)$



4. (6%) Consider an optimization model with a non-unimodal objective function, but with a convex constraint set. Assume that the origin is infeasible. If the Phase I model stops at a local optima with objective function value equal to π, what can we conclude? Assume that we have the same optimization model, but that a Big-M method is used and, as before, a local optima is reached where the sum of the artificial variables is π. What can we conclude?

5. (16%) The Pueblo Pager Company manufactures three pager models—purple with a chime sound, black with Bolero, and red with Jean-Luc Picard saying "engage". The profit from the sale of each model is 25, 30 and 35 dollars respectively. The weekly minimum production requirements are 20 for the purple model, 120 for the black model, and 60 for the red model. Each type of pager requries a certain amount of time for the manufacturing of component parts, for assembling, and for packaging. Specifically, a dozen units of purple require three hours for manufacturing, four hours for assembling, and one hour for packaging. The corresponding figures for a dozen units of black are 3.5, 5, and 1.5, and for a dozen units of red are 5, 8, and 3. During the forthcoming week, the company has available 120 hours of manufacturing, 160 hours of assembling, and 48 hours of packaging time.

a. Formulate this production scheduling problem as a linear programming model.
b. Does it make sense to use a linear program for this model? Why or why not?

6. Consider the following linear program.

$$\begin{aligned} \max\ p(\mathbf{x}) = 100x_1 + 90x_2 & \\ \text{s.t. } x_1 - x_2 &= 0 \\ 4x_1 + 2x_2 &\leq 520 \\ x_1 + x_2 &\geq 50 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(a) (4%) Is $\mathbf{x} = (0, 0)$ feasible? Why or why not?

(b) (8%) Construct the Phase I model in standard form if (0,0) is infeasible. Otherwise, construct the Phase II model in standard form.

7. A simplex search has been applied to a linear program. The search has reached iteration t which is summarized below.

max $c^T x$	500	450	0	0	0	b
	-3.5	5	1	0	0	60
	a	2	0	1	0	15
	b	0	0	0	1	8
	N	B	B	N	B	
x^t	0	7.5	22.5	0	8	3375
$\Delta x^{(2)}$	1	**c**	1	0	**d**	

(a) (6%) Determine values for **a**, **b**, **c**, and **d** which make $\Delta x^{(2)}$ an improving search direction.

(b) (6%) Determine values for **a**, **b**, **c**, and **d** which make $\Delta x^{(2)}$ an unbounded search direction.

8. The table below reflects the status of a simplex search for a linear program in standard form.

	x_1	x_2	x_3	x_4	x_5	
max **c**	3	5	0	0	0	**b**
	1	0	1	0	0	4
A	0	2	0	1	0	12
	3	2	0	0	1	18
	B	B	N	B	N	
$\Delta x^{(3)}$	-1	1.5	1	-3	0	
$\Delta x^{(5)}$	0	-0.5	0	1	1	

(a) (5%) What is the basic solution associated with the above table? Is it feasible? Explain.

(b) (8%) Which of the two directions are improving? Why?

(c) (5%) For the improving direction with the greatest rate of change compute the maximum step size λ.