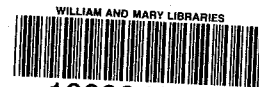


1284-1



1000949232

MATH 323 (Kincaid/Fall 1993)

Examination 1

September 30

XMATH 323

KINCAID

FALL '93 EXAM

COPI

Name: _____

1. The examination is closed book except for one page of 8.5 inch by 11 inch, hand written notes.
2. You may use a calculator, but other than this and your page of notes, you should have only writing instruments at your desk. (No trumpets, trombones, etc.)
3. Write with a soft pencil or a pen (blue or black ink).
4. Write only on the exam sheets provided.
5. Watch the blackboard or overhead for corrections and/or explanations.
6. Read carefully. Problems often have several parts or subsections.
7. The point distribution for each question out of 100 percent is shown in ().

Use in Library ONLY. 2 hr. Reserve.
\$1.00 per hr. overdue fine.

NOTICE: This Material
may be protected by copyright
law. (Title 17 US. Code)

1. Circle 'T' or 'F' according to whether each statement below is more nearly 'true' or 'false'. (18%)

- T F (a) The set of all points (x_1, x_2) satisfying $x_1^2 + x_2^2 \geq 2$ and $x_1 \geq 0$ is convex.
 T F (b) A feasible direction is improving for all values of the step size λ if the objective function is linear.
 T F (c) Linear Programs are, in general, more tractable than discrete optimization problems.
 T F (d) Linear objective function contours intersect the (non-empty) feasible region of an optimization model.
 T F (e) At $\mathbf{x} = (0, -1)$ in the feasible set defined by $-1 \leq x_1 \leq 1$ and $-1 \leq x_2 \leq 1$ the direction $(2, 1)$ is feasible.
 T F (f) If Phase I of a two-phase optimization procedure stops with a global minimum, then we conclude that the original (Phase II) problem is feasible.

2. (4%) Fill in each blank with one word so that the following statement is true.

A _____ objective function and _____ feasible set has the property that every local optimum is a global optimum.

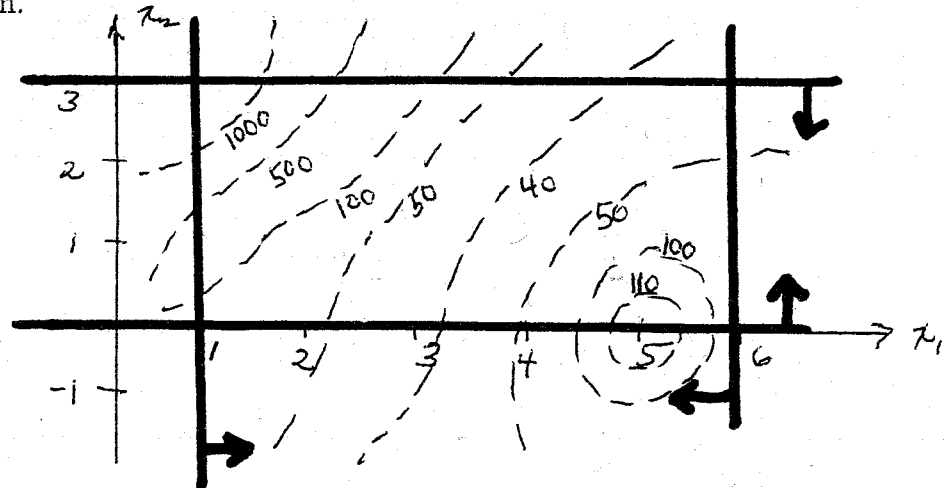
3. (8%) In the plot below, indicated whether each of the specified points is infeasible, feasible but not a local optimum, a local optimum, or a global optimum for a maximization problem.

(a) $(5, 0)$

(b) $(2, -1)$

(c) $(3, 2)$

(d) $(0, 0)$



4. (4%) Which of the following is (are) a feasible basis for a standard linear program with constraint matrix A.

$$\begin{pmatrix} 0 & 1 & 2 & 1 & 0 \\ 2 & 3 & -1 & 0 & 1 \end{pmatrix}$$

- (a) x_1 and x_2 (b) x_2 and x_3 (c) x_1 and x_5 (d) x_3 and x_4

5. (4%) A discrete improving search over $x_1, x_2,$ and $x_3 \in \{0, 1\}$ employs move set $\mathcal{M} = \{(1, 0, 0), (0, -1, 0), (0, 0, 1)\}$. Which of the following solutions is in the neighborhood of $\mathbf{x} = (0, 1, 1)$.

- (a) $(0, 0, 1)$ (b) $(0, 2, 1)$ (c) $(1, 1, 1)$ (d) $(0, 1, 2)$

6. (11%) The *Save the Bay* special interest group wants to restrict the amount of pollutants the power plant *AGLOW* is dumping into the James River. The pollutants—dilithium and radioactive waste—are to be restricted to 50 lbs/MG and 25 lbs/MG (MG = Million Gallons) respectively. *AGLOW* diverts a portion of the James River (to cool the reactor core), adds the pollutants to the water by using one of four available treatments, and sends the water back to the river. The characteristics of the four treatments are given below. Any combination of the treatments may be used. The river has a mass flow of 100,000 MG/day. In addition, you may assume (i) that the river water diverted to *AGLOW* is free of pollutants; (ii) the addition of pollutants does not affect the mass of the flowing water; and (iii) *AGLOW* has to process *at least* 1,000 MG of river water per day. Let w_i denote the MG of water processed by *AGLOW* each day using treatment i . Hence, the objective is to *minimize* $100w_1 + 120w_2 + 105w_3 + 115w_4$. Formulate the linear constraints to determine the amount of water to be processed by each of the four treatments so that the total cost is minimized.

TREATMENT	1	2	3	4
dilithium	10	15	12	17
rad waste	7	5	6	4
Cost/MG	100	120	105	115

7. Consider the following linear program.

$$\begin{aligned} \max \quad & c(\mathbf{x}) = 0.4x_1 + 0.5x_2 \\ \text{s.t.} \quad & 0.3x_1 + 0.1x_2 \leq 2.7 \\ & 0.5x_1 + 0.5x_2 = 6 \\ & 0.6x_1 + 0.4x_2 \geq 6 \\ & x_1, x_2 \geq 0 \end{aligned}$$

(b) (a) (8%) Construct the artificial model needed to begin a Phase I search.

(c) (b) (4%) Is $\mathbf{x} = (0, 0)$ feasible? Why or Why not?

8. A simplex search has been applied to a *maximize* linear program with objective function $c(\mathbf{x}) = 3x_1 + 5x_2 - x_3$. The simplex search has reached iteration t which is summarized below.

	N	B	N	B	N	B
$x^{(t)}$	0	2	0	1	0	3
$\Delta x^{(1)}$	1	-2	0	-1	0	0
$\Delta x^{(3)}$	0	1	1	0	0	2
$\Delta x^{(5)}$	0	1	0	-2	1	-4

(a) (6%) Determine which of the three simplex directions are improving. (There may be more than one.)

- (b) (7%) For one improving direction compute the maximum step size λ and, if λ is finite, compute the new basic solution.

9. The table below reflects the status of a simplex search for a linear program in standard form.

	x_1	x_2	x_3	x_4	x_5	
max c	3	5	0	0	0	b
	1	0	1	0	0	15
	0	2	0	1	0	8
	3	2	0	0	1	60
	N	N	B	B	B	

- (a) (3%) What is the basic solution associated with the above table? Is it feasible? Explain.


- (b) (7%) Compute all simplex directions at this solution.

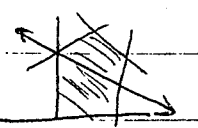
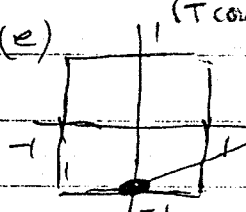
10. (16%) Consider the discrete optimization problem given below.

$$\begin{aligned} \max \quad & f(\mathbf{x}) = 9x_1 + 5x_2 + 6x_3 + 4x_4 \\ \text{s.t.} \quad & 6x_1 + 3x_2 + 5x_3 + 2x_4 \leq 15 \\ & x_3 + x_4 \leq 1 \\ & -x_1 + x_3 \leq 0 \\ & -x_2 + x_4 \leq 0 \\ & x_1, x_2, x_3, x_4 \in \{0, 1\} \end{aligned}$$

Begin from $\mathbf{x} = (1, 0, 0, 0)$ and perform a local improvement search with move set $\mathcal{M} = \{(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)\}$ until a local optimal solution is reached. If, at any stage, two moves are possible choices, then take the one with the best objective function value. Please show all your work and carefully label each step.

Exam 1 (key)

1) (a)  F (b) F, constraints may restrict choice of λ (T could be argued as well) (c) T, no general algorithm to produce global opt. that is reasonable.

(d) T  (e)  T, if λ is restricted to say $\epsilon < 1/4$.

(f) F, global min may have art. var. > 0 .

- 2.) unimodal, convex linear, unconstrained
- (3) (a) local optimum, feasible (b) infeasible
- f) a, b, d, check 2×2 determinant (c) feasible, not locally optimal
- g) a, c, d. (d) infeasible

6)
$$\min 100w_1 + 120w_2 + 105w_3 + 115w_4$$

$$\text{s.t. } w_1 + w_2 + w_3 + w_4 \geq 1000$$

$$w_1 + w_2 + w_3 + w_4 \leq 100000$$

$$10w_1 + 15w_2 + 12w_3 + 17w_4 \leq 50 (w_1 + w_2 + w_3 + w_4)$$

$$7w_1 + 5w_2 + 6w_3 + 4w_4 \leq 25 (w_1 + w_2 + w_3 + w_4)$$

$$w_1, w_2, w_3, w_4 \geq 0$$

(7) (a) $x = (0, 0)$ is not feasible. Constraints (2) + (3) are violated

(b)
$$\min x_3 + x_4$$

$$\text{s.t. } .3x_1 + .6x_2 \leq 2.7$$

$$.5x_1 + .5x_2 + x_3 = 6$$

$$.6x_1 + 1.4x_2 + x_4 \geq 6$$

(c)
$$C \cdot \Delta x^{(1)} = -7x$$

$$C \cdot \Delta x^{(2)} = 4 \quad \checkmark \text{ improving}$$

$$C \cdot \Delta x^{(3)} = 5 \quad \checkmark$$

(d)
$$x^{(1)} = (0 \ 2 \ 0 \ 1 \ 0 \ 3) + \lambda (0 \ 1 \ 1 \ 0 \ 0 \ 2)$$

(9) (a) $\vec{x} = (0, 0, 15, 8, 60)$ is the basic soln. $\Rightarrow \lambda = \infty$, unbounded

It is feasible since all its components are ≥ 0 $x^{(1)} = (0 \ 2 \ 0 \ 1 \ 0 \ 3) + \lambda (0 \ 1 \ 0 \ -2 \ 1 \ -4)$

non-negative (or plug into original equations) $\lambda = \min \{1/2, 3/4\} = 1/2$

(b)
$$\Delta x^{(1)} = (1 \ 0 \ -1 \ 0 \ 3) \quad \Delta x^{(2)} = (0 \ 1 \ 0 \ -2 \ -2) \quad x^{(1)} = (0, 2^{1/2}, 0, 0, 1/2, 1)$$

(10)
$$x^{(0)} + (1000) = (2000) \quad \text{I} \quad -$$

$$+ (0 \ 1 \ 0 \ 0) = (1100) \quad \text{F} \quad 14$$

$$+ (0 \ 0 \ 1 \ 0) = (1010) \quad \text{F} \quad 15$$

$$+ (0 \ 0 \ 0 \ 1) = (1001) \quad \text{I} \quad -$$

$$x^{(1)} + (1000) = (2010) \quad \text{I}$$

$$+ (0 \ 1 \ 0 \ 0) = (1110) \quad \text{20 F} \quad \leftarrow$$

$$+ (0 \ 0 \ 1 \ 0) = (1020) \quad \text{I}$$

$$+ (0 \ 0 \ 0 \ 1) = (1011) \quad \text{I} \quad + \dots$$