Math 323: Exam 2 Key—Fall 2007

Operations Research I–Deterministic Models

1. (a) False. Replace 'less and less' with *more and more.*

1. (b) True. The primal is a maximization problem. Each primal $\geq$ constraint implies that the associated dual variable is ≤ 0.

1. (c) True. The word *may* is critical. I also accepted an answer of False with 'strictly less than' replaced by $\leq$.

1. (d) False. Replace $\geq$ with $\leq$.

2. (a) $\Delta x_{leave} = -3$ since x_3 leaves the basis. x_3 is associated with column one of the current B^{-1}. As a result the entries in column 1 of E will be $e_{11} = -1/-3$ and $e_{22} = -(-4/-3)$. Column 2 of E is identical to the second column of I_2.

2. (b) Recall that by solving $A\vec{\Delta x} = \vec{0}$ we get $\Delta\vec{x}_B = -B^{-1}\vec{a}_k$ for all k nonbasic. Now $\vec{c}^T\vec{\Delta x}^k \leq \vec{0}$ is equivalent to $\vec{c}_B^T\Delta\vec{x}_B + c_k \leq \vec{0}$ for all k nonbasic. Substituting in for $\Delta\vec{x}_B$ we get $\vec{c}_B^T(-B^{-1}\vec{a}_k) + c_k \leq \vec{0}$. Rearranging and substituting $\vec{v} = \vec{c}_B^T B^{-1}$ we get $\vec{v}\cdot\vec{a}_k \geq c_k$. Now recall that the dual constraints are given by $A^T v \geq \vec{c}$. Thus, for each nonbasic variable x_k the corresponding dual constraint is $\vec{v}\cdot\vec{a}_k \geq c_k$.

2. (c) A negative dicycle is a directed cycle whose edge cost sum is a negative number. The Bellman-Ford algorithm is restricted to graphs that have no negative dicycles (it also can be used to detect the presence of negative dicycles). Dijkstra's algorithm is restricted to graphs with non-negative edge costs. All graphs with non-negative edge costs have no negative dicycles but the converse is not true. That is, the exclusion of negative dicycles does not preclude the presence of negative cost edges. Hence, the Bellman-Ford algorithm admits a larger class of graphs.

2. (d) Circle optimal (all $\bar{c}_k \leq 0$), alternate optimal ($\bar{c}_k = 0$ for nonbasic variable x_5) degenerate (a basic variable has a value of zero) and dual feasible.

3. (a) The current objective function value is \$775, 833.3.

3. (b) The largest increase for T is 500. This information can be found under the righthand side ranges.

3. (c) Draw the graph. x-axis denotes T. y-axis denotes objective value. Slope is 163.33. Label points $(0, 775, 833.3)$ and $(500, 775, 833.3 + 500 * 163.33)$.

3. (d) Let $T = 500 + \epsilon$, for some $\epsilon > 0$.

4. (a) Compute $\bar{c}_j = c_j - c_B^T \cdot B^{-1} \cdot a^j$ for $j = 4$ and $j = 5$, the nonbasic variable indices. Be sure to use the correct order for the basic variable components of c_B^T. $c_B^T \cdot B^{-1} = (0, 1/5, 7/5)$. Then $\bar{c}_4 = -1/5$ and $\bar{c}_5 = -7/5$.

4. (b) $x^{(2)}$ is optimal since both $\bar{c}_4$ and $\bar{c}_5$ are less than 0. No nonbasic variable can enter the basis and further increase the objective function value.

4. (c) The complementary dual solution is $v = (0, 1/5, 7/5)$. It is optimal. The stopping condition for the simplex search is to check for dual feasibility. Thus the complementary dual solution is feasible and its dual objective function value is equal to the primal optimal objective function value. Strong duality tells us that this dual solution must be optimal.

5. (a) The marginal value (dual variable value) of scrap 1 is 3.38 kr/kg. Moreover, this marginal value is valid over scrap 1 amounts between 60.42 and 83.33. The increase from 75 to 78 lies within this range. Consequently, the new objective value is $-9,953 + 3.38(3) = -9,943.56$. An increase from 75 to 88 lies outside the range. Hence, we can only provide upper and lower bounds on the change in the objective function value. The lower bound is given by 3.38(8.33) while the upper bound is given by 3.38(13).

5. (b) The price at which chromium will enter the basis is $-60 + 13.38 = -46.62$.

5. (c) The dual objective is a maximization. The dual constraints are all $\leq$ type. v_1 is free (URS). v_2, v_4, v_6, and v_8 are all ≥ 0. The remaining v_j are ≤ 0.

5. (d.) The new dual constraint is $v_1 + .006v_2 + 006v_3 + .02v_4 + .02v_5 + .04v_6 + .04v_7 + .002v_8 + .002v_9 \leq c_{new}$. Substituting in the optimal values for the dual variables we get $-9.8889 + 0.00132 - .0076 - .0144 - .00086 = -9.910$. Hence, the new scrap heap will enter the basis when the price is 9.91 kr/kg.

6. $v[5] = \min\{v[2] + c_{25}, v[3] + c_{35}, v[4] + c_{45}\} = \min\{2 + 1, 3 - 1, 5 + 3\} = 2$.