

Math 307 Exam 2 Review

Vocabulary, examples, and theorems from the first review sheet, plus the following additional material:

the alternating group A_n ; the order of A_n ; left (right) coset of a subgroup H of a group G ; for left cosets aH, bH , either $aH \cap bH = \emptyset$ or else $aH = bH$; all cosets of H have the same size; Lagrange's theorem; When is the converse of Lagrange's theorem true? (yes for finite Abelian groups, no for finite groups in general); direct product and direct sum $G \times H$ of two groups; $\mathbb{Z}_m \times \mathbb{Z}_n$ is isomorphic to \mathbb{Z}_{mn} if and only if $\gcd(m, n) = 1$; finitely generated group; Fundamental theorem for finitely generated Abelian group; examples of groups that are/are not finitely generated; decomposable and indecomposable groups; if G is a finite abelian group and m is a divisor of $|G|$, then G has a subgroup of order m ; why is $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3$ not isomorphic to $\mathbb{Z}_4 \times \mathbb{Z}_9$; homomorphism; for a homomorphism $f : G \rightarrow H$, $f(e_G) = f(e_H)$ and $f(a^{-1}) = (f(a))^{-1}$; if $f : G \rightarrow H$ is a homomorphism, and if K is a subgroup of G , then $\{f(x) : x \in K\}$ is a subgroup of H (also if L is a subgroup of H , then $\{x \in G : f(x) \in L\}$ is a subgroup of G); the kernel of a homomorphism; a homomorphism is 1-1 if and only if its kernel has exactly one element; normal subgroup; equivalent definitions of a normal subgroup; kernel of a homomorphism is a normal subgroup; if N_1 and N_2 are normal subgroups of G , then so is $N_1 \cap N_2$; which subgroups of an abelian group are normal?; example of a non-normal subgroup of a group; equivalent versions of normality; the quotient group (= factor group) construction G/N where N is a normal subgroup of G ; the operation in G/N is well-defined; factor group is a synonym for quotient group; fundamental theorem for group homomorphisms; A_4 has no subgroup of order 6; introductory material on rings, as covered up to the end of Monday, March 29.