Math 108: Notes on Elasticity of Demand

Percentage increases, Percentage rate of increase

Some time ago, Coca-cola in the second floor vending room of this building cost 75 cents per can. Then the price jumped to $1 per can. The wrong way to look at this situation is "Twenty-five cents is not much money, so the increase was small. " The right way to look at the situation is "Dividing the increase by the former price shows a 33% price increase and that's big." It's the percentage increase, not the actual increase, that should worry consumers.

Whenever you have a function $y = f(x)$ relating two quantities, we know that $f'(x)$ gives the rate of change of y compared to x. The

$$\text{relative rate of change} = \frac{f'(x)}{f(x)} \text{ and percentage rate of change} = 100 * \frac{f'(x)}{f(x)}.$$

Note that these quantities probably depend on the x value you started with and should really be called the relative and percentage rates of change starting at x.

Elasticity of Demand

Demand for gizmos is sensitive to unit price. An increase in price causes a drop in demand. Suppose we have a demand equation $x = f(p)$ where p is unit price and x is the number of gizmos that can be sold at price p. The *percentage rate of change in demand* is

$$100 * \frac{f'(p)}{f(p)}.$$

This predicts the percentage change in demand corresponding to a price increase of $1, and it is always negative. Starting at a given price p, raising prices by $1 will result in a percentage increase in price of $100 * \frac{1}{p}$. Economists define *elasticity of demand starting at price level p* to be the ratio

$$E = \left| \frac{\text{percentage change in demand}}{\text{percentage change in price}} \right| = \left| \frac{100 * \frac{f'(p)}{f(p)}}{100 * \frac{1}{p}} \right| = \left| \frac{p * f'(p)}{f(p)} \right|.$$

Why use the absolute value? Because otherwise every entry in a table of demand elasticities would be negative. Note that because our $f'(p) < 0$, this is exactly the same as the book's definition $E = \frac{-p*f'(p)}{f(p)}$.

Why Care About Elasticity of Demand?

Recall that our revenue (= income) for selling gizmos is given by

$$R = \text{Revenue} = \text{unit price} * \text{number sold} = p * f(p)$$

where p is unit price and $f(p)$ is the demand function. Our real interest is in the question "Will our revenue rise if we increase prices slightly, starting at price level p?" In other words, is R an increasing function near p? In still other words, is $R'(p) > 0$?

We use the product rule to find $R'(p)$ and throw in some algebraic trickery to show how elasticity of demand, E, is the key. At one point we will use the fact that $\frac{p*f'(p)}{f(p)} = -E$. Here is the calculation:

$$R' = p * f'(p) + 1 * f(p) = f(p) * \left(\frac{p * f'(p)}{f(p)} + 1 \right) = f(p) * (-E + 1).$$

Now $f(p) > 0$ always so that whether $R'(p) > 0$ boils down to whether $1 - E > 0$ and that is the same as whether $1 > E$. We now have:

Theorem: $R'(p) > 0$ if and only if $E < 1$ where $E = \left|\frac{p*f'(p)}{f(p)}\right|$ so that

- if $E < 1$ then a slight price rise will cause a revenue increase, and
- if $E > 1$ then a slight price rise will cause a revenue decrease.

Economists say that *demand is elastic* at price level p if $E > 1$ at price p, and *demand is inelastic* at price level p if $E < 1$. Therefore our theorem says "A slight price rise in an inelastic demand situation will cause a revenue rise while a slight price rise in an inelastic demand situation will cause a revenue drop." In other words, if demand is inelastic, then price and revenue move in the same direction, and if demand is elastic, then price and demand move in opposite directions.

Example

Suppose our demand function is $f(p) = \frac{10{,}000}{p+50} - 30$ and the current unit price is $p = 150$. Is demand elastic or inelastic? What will be the effect on revenue of a slight price increase?

We see that $f'(p) = \frac{-10{,}000}{(p+50)^2}$ so that

$$E = \left|\frac{p * \frac{-10{,}000}{(p+50)^2}}{\frac{10{,}000}{p+50} - 30}\right|.$$

Substituting $p = 150$ and doing lots of arithmetic gives $E = \frac{15}{8} > 1$ so that demand is elastic at price level $p = 150$ and raising prices slightly from $p = 150$ will cause revenue to drop.

More interesting is the conclusion that lowering prices slightly from the $p = 150$ level will cause revenue to rise.

On the other hand, starting at price level $p = 10$, $E = \frac{25}{123} < 1$ so that demand is inelastic, so that a slight price rise from $p = 10$ will cause a revenue increase.

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