Page 207, #11: The Hamburger Problem

1) The demand curve: A straight line demand curve contains the points $(p, q) = (2, 10000)$ and $(p, q) = (2.40, 8000)$ where p = unit price and q = number sold. The slope of this line is $\frac{8000-10000}{2.40-2} = -5000$ so the equation must be $q = mp + b = -5000p + b$. Because $(2, 10000)$ is on the demand line we have $10000 = -5000 * 2 + b$ so that $b = 20000$ and therefore the demand equation is $q = -5000p + 20000$.

2) Revenue is unit price times number sold, so

$$Rev = pq = p(20000 - 5000p) = 20000p - 5000p^2.$$

To maximize Rev, we begin by solving $Rev' = 0$, that is $20000 - 10000p = 0$, and we get $p = 2$ as a candidate for the price that gives maximum or minimum revenue. To determine which, we look at concavity of the revenue function, using $Rev'' = -10000$. Because $Rev''(2) = -10000 < 0$ we know that $p = 2$ gives a maximum value for Revenue.

3) Profit is always "Revenue minus Cost". Costs come in two varieties – fixed costs (which do not depend on the number sold) and variable costs that are determined by the number q sold. In this problem, each hamburger costs us \$.60 so that our variable hamburger costs will be $.6q$. We also have fixed costs of \$1000 so that our cost formula is $Cost = 1000 + .6q$. Therefore our profit formula is

$$Profit = Rev - Cost = (20000p - 5000p^2) - [1000 + .6(20000 - 5000p)] = 23000p - 5000p^2 - 13000.$$

Note that we used the demand equation to substitute for the variable q in the cost formula.

To maximize profit we begin by solving $Profit' = 0$, that is $0 = 23000 - 10000p$. We get $p = 2.30$ as our candidate for maximum or minimum profit. To determine which, we determine the concavity of the Profit curve at $p = 2.30$ by using $Profit'' = -10000 < 0$ so we see that $p = 2.30$ gives a maximum profit.

4) Think back to the theorem about how much of a price increase to pass on to customers in a situation where we have a straight line demand curve. As you can see from the example and proof of the theorem, we were talking about an increase in the cost of making each gizmo that we sell, i.e., an increase in unit cost.

How does the Hamburger Problem fit with our earlier theorem? Think of the two scenarios. In the first we have fixed costs of \$1000 and no variable costs (perhaps because somebody gives us the hamburger free). In that case our profit is given by $Profit_1 = Rev - 1000 = -5000p^2 = 20000p - 1000$. We find that our optimal price in the first scenario is $p = \$2$. Suddenly our supplier raises the price of hamburger meat from zero to \$0.60 per burger. That is a 60 cent increase in unit cost, and because we have a straight line demand curve, our theorem says "Pass on half of the sixty cent cost increase to our customers." In other words, we should increase our price from the previously optimal level of \$2 to \$2.30, just as we worked out in part (3) above.