

## What to Do About Price Increases for Supplies

**Example:** Suppose we have the demand curve  $q = f(p) = 1170 - 2p$  for the gizmos that we make, and that each gizmo costs us \$20 to produce. Then our revenue function is

$$Revenue = pq = p(1170 - 2p)$$

and our cost of producing  $q$  many gizmos is

$$Cost_1 = 20q = 20(1170 - 2p).$$

Consequently, our profit function is

$$Profit_1 = Revenue - Cost_1 = p(1170 - 2p) - 20(1170 - 2p) = 1170p - 2p^2 - 23400 + 40p.$$

This curve is a parabola opening downwards, so we can find the optimal price (= the price that will give us maximum profit) by solving

$$0 = (Profit_1)' = 1170 - 4p + 40 = 1210 - 4p$$

and we find that optimal price in this first scenario is  $OptimalPrice_1 = \frac{1210}{4} = 302.50$ .

Now suppose our supplier raises prices so that each gizmo costs us \$30 to make, rather than \$20. What should we do? Should we pass on the price increase to our customers, charging them \$312.50 per gizmo? Or should we absorb the price increase, and continue charging \$302.50 per gizmo even though that will eat into our profits?

To decide, let's rework the problem above, using the new cost figure of \$30 per gizmo produced, and find the new optimal price. The demand equation doesn't change, so the revenue formula is the same. However, the cost function is changed, to  $Cost_2 = 30q = 30(1170 - 2p)$  and therefore the profit formula is changed to

$$Profit_2 = p(1170 - 2p) - Cost_2 = p(1170 - 2p) - 30(1170 - 2p) = 1170p - 2p^2 - 35100 + 60p.$$

This formula is also a parabola opening down, so we can find the optimal price if we solve

$$0 = (Profit_2)' = 1170 - 4p + 60 = 1230 - 4p$$

which gives us  $OptimalPrice_2 = \frac{1230}{4} = 307.50$ .

What do we conclude? We should not pass on the entire \$10 per gizmo price increase to our customers, and we should not absorb the price increase ourselves. Instead, if our goal is to maximize profits, we should pass on \$5 of the \$10 price increase to our customers, i.e., we should pass on half of the price increase.

What conclusion does that example suggest? Perhaps "Pass on half of the price increase" always works, or perhaps it's only works if the cost rises by 50% (from \$20 to \$30 per gizmo) or perhaps something else. Here is the real conclusion.

**Theorem:** Suppose the demand curve for our product is linear and that our suppliers raise their prices to us by \$I per gizmo. In order to maximize profits, we should increase our prices to our customers by  $\$ \frac{I}{2}$  per gizmo.

Proof: Our demand curve looks like  $q = mp + b$  for certain fixed values of the parameters  $m$  and  $b$ , and we note that  $m < 0$ . Consequently our revenue formula is  $Rev = pq = p(mp + b)$ . Consider the case where our supplier charges us \$K for the material needed to make one gizmo. Then our cost function is  $Cost_1 = Kq = K(mp + b)$  so that our profit function is

$$Profit_1 = p(mp + b) - K(mp + b) = mp^2 + bp - Kmp - Kb.$$

This is a parabola opening downward because  $m < 0$ , so the optimal price for us to charge is found by solving

$$0 = (Profit_1)' = 2mp + b - Km$$

whose solution is  $OptimalPrice_1 = \frac{Km - b}{2m}$ .

Now consider a second scenario, where each gizmo costs us  $K + I$  to make. The demand curve doesn't change, so the revenue function doesn't change, but the cost function changes to  $Cost_2 = (K + I)q = (K + I)(mp + b)$ . Therefore

$$Profit_2 = p(mp + b) - (K + I)(mp + b) = mp^2 + bp - (K + I)mp - (K + I)b.$$

Again, this is a parabola opening downwards because  $m < 0$  so we can find the new optimal price by solving

$$0 = (Profit_2)' = 2mp + b - (K + I)m$$

which gives

$$OptimalPrice_2 = \frac{(K + I)m - b}{2m} = \frac{Km - b}{2m} + \frac{Im}{2m} = p_1 + \frac{I}{2}.$$

Therefore we should increase the price that we charge per gizmo from the old level of  $OptimalPrice_1$  to the new level of  $OptimalPrice_2 = OptimalPrice_1 + \frac{I}{2}$ .  $\square$