Maximizing Revenue: Problem 45, page 173

Revenue (or income) can be computed in several ways, depending upon the application being considered. In one situation, revenue is (price per item)*(number of items sold). In the scenario considered in this problem, a mutual fund company charges an advertised rate as a service fee and revenue is (decimalized fee percentage)*(total assets of the fund).

Maximizing revenue involves a tricky balancing act. A higher price per item would seem to increase revenue, but it also tends to cause lower total sales and the combined impact on revenue is not clear. A higher advertised rate would seem to increase revenue to the mutual fund company, but it also tends to cause the public to invest less in the mutual fund, shrinking the fund's asset base. Luckily, we have calculus to solve the revenue maximization problem.

In this problem, $x =$ the advertised percentage service fee charged by a mutual fund and $A(x) =$ fund assets in billions if the fund charges x percent. Intuition says that as x increases, $A(x)$ decreases.

a) Suppose that history convinces us that $A(x)$ is a straight line formula, $A(x) = mx + b$, and that the points $(0.47, 41)$ and $(0.18, 300)$ are known to lie on the line. Then

$$\textit{slope } m = \frac{300 - 41}{0.18 - 0.47} = -\frac{259}{0.29} = -\frac{25900}{29}$$

which is roughly -893.1034483. Because $(0.18, 300)$ is on the line, $300 = m * (0.18) + b$ and we get $b = 300 + \frac{259*18}{29}$ which is roughly 460.7586287. Now we know the formula for $A(x)$ is roughly $A(x) = -893.103x + 460.759$.

b) In this case, revenue is (decimalized percentage of assets) times (assets). Recall that to decimalize 0.18 percent, divide by 100 to get .0018. This allows us to compute revenue generated when (for example) a service charge of 0.18 percent is imposed: we simply evaluate $0.0018 * A(0.18)$.

c) The formula for revenue is (decimalized service fee percentage)*(total assets) so if we charge x percent and use rounded off numbers we get

$$R = \left(\frac{x}{100}\right) * (-893.103x + 460.759) = \frac{1}{100} * \left(-893.103x^2 + 460.759x\right).$$

To maximize the function R we begin by solving

$$0 = R' = \frac{1}{100} * (-893.103 * 2 * x + 460.759)$$

giving a candidate x value of $x = \frac{470.759}{2*893.103}$ which is roughly 0.257954009. To determine whether this candidate gives a maximum value for R, we look at the sign of R' on opposite sides of the candidate and see that the x candidate that we found gives a maximum value for R.

In a few days you will learn another technique for testing candidates. When we compute the second derivative, we see $R'' = \frac{1}{100} * (-893.103) * 2 < 0$. Because our candidate $x = \frac{470.759}{2*893.103}$ causes $R' = 0$ and $R'' < 0$, we have

(R has a horizontal tangent) plus (R is concave down)

so we have a maximum value for R at the candidate value $x = 0.257954009$.