# Index

#### А

accelerated life model, 169-172, 345-356 covariates, 170 exercises, 182-184, 367-370 exponential baseline, 346-355, 367-370 inference, 345-356 link function, 170 matrix formulation, 344-345 references, 177, 366-367 two-sample case, 346, 348-352 Weibull baseline, 355-356, 369 accelerated life test, 285-286, 312 actuarial science, 1, 2 adjusted Kolmogorov-Smirnov test statistic, 436 age replacement policy, 187-188 age-specific death rate, 63 Agresti-Coull confidence interval, 378-379 air conditioning failure times, 413 air conditioning times between failures, 273, 336 aircraft component failure times, 336, 410 airline safety, 7-10, 17 alternating renewal process, 205-211, 230-231 Anderson-Darling goodness-of-fit test, 440 arctangent distribution, 136, 154, 438 Arrhenius model, 324, 329 asymptotic properties of the likelihood function, 254-257 asymptotically unbiased estimator, 235 automotive a/c switch failure times, 285, 296-299, 306-307, 328, 331, 334, 425-426 availability, 13, 205-211 average availability, 206-211 average failure rate, 106

## B

backward stepwise regression, 371 ball bearing failure times, 277, 285, 287–289, 314–317, 328, 339, 376–379, 420–423, 427–430

Barnett, Arnold, 8 bathtub-shaped hazard function, 11, 65-67, 85, 89, 98, 110, 128-130, 143, 157, 179, 325 Bayesian credible interval, 378 Bayesian models, 135, 168, 329, 336 Bayesian reliability demonstration test, 312 Benford distribution, 95 Benford leading digit data, 433 Benford's Law, 433 Benford, Frank, 433 Bennetts' method, 46 Bernoulli distribution, 132-135, 376 Bernoulli, Daniel, 157 beta distribution, 132-135, 136, 153, 378 beta hazard rate distribution, 136 beta-modified Weibull distribution, 136 bi-Weibull distribution, 179 biased estimator, 235, 331 bimodal failure, 58-59 binary assumption, 21 binomial distribution, 96, 375-376 biostatistics, 1, 2 Birnbaum imporance, see structural importance Birnbaum–Saunders distribution, 136 birth-death model, 218-222 bivariate exponential distribution, 137, 155 block diagram, 21-29 bridge system, 25 k-out-of-n system, 23–24 parallel system, 23 series system, 22-23 block replacement policy, 187-188 Breslow, Norman, 363 bridge system, 25 Brinell hardness number, 370 burn-in, 85, 94, 97, 106, 145, 189-190, 331 burn-in failure times, 331

Burr distribution, 137

Burr system of distributions, 137 Burr Type XII distribution, 150

## С

calculation of system reliability, see reliability censoring, 13-14, 257-265 alignment, 261–262 current status data, 260, 339 interval, 119, 260, 335 left, 259-260, 280, 330 likelihood function, 263 notation, 261-263 progressive, 153, 267, 334 proportional hazards, 361-363 right, 258-260 random, 259, 302-305, 361-363, 437 Type I, 259, 300-301, 330-333 Type II, 258–259, 294–300, 330, 331 taxonomy, 260 central limit theorem, 248, 254, 268, 269 Certified Reliability Engineer Exam, 19 Challenger accident, 1, 5-7, 17, 281 characteristic function, 61 characteristic life, 120, 128 Chebyshev's inequality, 86 chi-square distribution, 125, 132-135, 254, 302-306, 308-310 chi-square test, 415-419, 433 circular k-out-of-n system, 45-46, 86 clinical trial, 285 Clopper-Pearson confidence interval, 378-379 coefficient of variation, 74, 114, 121, 125, 132-133, 182, 193-194 coherent system, 25-27, 48-51 cold standby system, see standby system competing risks, 12, 156-165, 176, 452-454 and finite mixtures, 168 crude lifetimes, 158-165, 177-178 exercises, 177-180, 275, 331, 342 in Markov models, 214-215, 217, 220 net lifetimes, 157-165, 177-180 complete data set, see censoring component vs. system redundancy, 26-27, 45 compound distribution, see mixtures conditional intensity function, 200-201, 202-204 conditional survivor function, 12, 62, 79-81 confidence interval, 240-247 actual coverage, 377

approximate, 243-244, 300-305, 351 asymptotically exact, 244 conservative, 379 definition, 240 exact, 241, 349 expected width, 244 exponential population, 115, 242-243, 289, 293-298, 301-307, 331-336 for *S*(*t*), *376–385*, 406 for the cumulative intensity function, 394 halfwidth, 241 interpretation, 243 joint distribution of bounds, 244-247 one-sided, 241, 333 scatterplot, 245-247 stated coverage, 240 two-sided, 241 variance of the width, 245 confidence region, 316, 320, 329, 341 consecutive k-out-of-n system, 45 consistency, 239, 376 consumer's risk, 307-308 continuous lifetimes, see lifetime distribution representations continuous mixtures, 168-169 continuous-time Markov chain, 211 convolution property, 125 copy machine failure times, 396-398, 414, 430-431 corrective maintenance, 188 counting function, 191 counting process, 191 covariates, 15, 169-175, 344-366 Cox, David, 173, 385 Coxian distribution, 136 Cramér-Rao inequality, 237-238, 266, 269 Cramér-von Mises goodness-of-fit test, 440 crude lifetimes, 158-165, 177-178 Cullen and Frey graph, 135 cumulative damage model, 137, 225 cumulative hazard function, 67, 70 cumulative intensity function, 197, 224, 226, 392-406 current life table, *see* life table current status data, see censoring cut vector, 30 cut vector technique for calculating system reliability, 39-40

#### D

data analysis, 13-16 data sets air conditioning failure times, 413 air conditioning times between failures, 273 aircraft component failure times, 336 automotive a/c switch failure times, 285 ball bearing failure times, 285 Benford leading digit data, 433 burn-in failure times, 331 copy machine failure times, 397 diesel engine fan failure times, 413 digits of  $\pi$ , 417 electric generator failure times, 409 electrical insulating fluid failure times, 411 electronic device failure times, 286 equipment failure times, 412 heat pump compressor failure times, 400 horse kick data, 417 integrated circuit failure times, 412 jet engine failure times, 437 leukemia nonmaintained group survival times, Euler's reflection formula, 450 411 leukemia remission times, 285, 412 leukemia survival times, 372 lung cancer survival times, 369 minimum wage data set, 325 rat survival times, 271 snubber failure times, 410 Space Shuttle failure times, 281 turbine wheel failure times, 339 U.S.S. Halfbeak engine failure times, 392 wooden beam failures, 371 decomposition technique for calculating system reliability, 40-41 definition of reliability, 2-5 definition technique for calculating system reliability, 35-36 degenerate distribution, 94 delta-star method, 46 density quantile function, 85 deteriorating item, 190 DFR (decreasing failure rate), 81–85, 152 DFRA (decreasing failure rate on the average), 85 diesel engine fan failure times, 413 digits of  $\pi$  data set, 416–417

discrete lifetimes, see lifetime distribution representations discrete Weibull distribution, 95 distribution classes, 81-85, 130, 342 DMRL (decreasing mean residual life), 149 Drenick's theorem, 223 dual system, 49-50 dwell time, see holding time

#### Е

efficient estimator, see minimum-variance unbiased estimator Efron, Bradley, 363 electric generator failure times, 409 electrical insulating fluid failure times, 411 electronic device failure times, 286, 317-325, 328, 352-355 empirical survivor function, 14-16, 288 equipment failure times, 412 Erlang distribution, 125, 132-135, 148, 193, 195, 235, 239, 243, 255, 443, 446 excess kurtosis, 135 expectation technique for calculating system reliability, 36-39 explanatory variables, see covariates exponential distribution, 111-119, 286-312 asymptotic properties, 254-255 comparing two, 305-307 complete data set, 287-294 confidence interval for  $\lambda$ , 115 confidence interval for  $\theta$ , 242–243, 245–247 consistency, 239 distribution classes, 130 distribution representations, 111, 126-127 exercises, 137-145, 330-339 Fisher information matrix, 250–252 fractiles, 74-75 in a renewal process, 195 in competing risks, 159–165 in continuous mixtures, 168-169 in finite mixtures, 167 life testing, 307–312 maximum likelihood estimator, 287, 295, 300, 302 mean residual life function, 68 memoryless property, 11-12, 111-112, 441

minimum-variance unbiased estimator, 237-238 moment ratio diagrams, 132-135 moments, 74-75, 114 order statistics, 115-117 probability plot, 291-293 properties, 111-119, 441-449 random right censoring, 302-305 references, 136, 328 relative efficiency, 236-237 score vector, 250-252 self-reproducing property, 115, 443 shifted, 271, 276 sums, 125 test for, 428-432, 439 Type I right censoring, 300-301 Type II right censoring, 294–300 unbiased estimator, 234-235 unit, 113, 442 exponential power distribution, 128, 130, 132-136, 150, 228, 341, 342 exponential-exponential distribution, 181 exponentiated distribution, 136 exponentiated Weibull distribution, 329, 366 extreme value distribution, 117, 123, 136, 154, 459-460

# F

*F* distribution, *306–307*, 378, 429–431 failure modes and effects analysis, 18, *45* failure replacement policy, 187–188 field joint rotation, 5–7 finite mixtures, 166–168 Fisher information matrix, 248–253 accelerated life model, *346–348*, 367 exponential( $\lambda$ ) population, 289, 295 Fisher, Ronald, 416 force of mortality, 63 forward stepwise regression, 371 Fréchet distribution, 136, 282 fractiles, 74–75

## G

gamma distribution, 123–125 convolution property, 125 distribution classes, 130 distribution representations, *123–125*, 127 exercises, 147–148, 336, 342

in a renewal process, 196, 229 in continuous mixtures, 169 moment ratio diagrams, 132-135 moments, 125 references, 136, 329 relationship to Weibull distribution, 125 special cases, 125 gamma function, 114, 450-451 gamma-Poisson distribution, 169 Gauss' multiplication theorem, 451 generalized F distribution, 137 generalized gamma distribution, 137 generalized inverse Weibull distribution, 136 generalized lambda family of distributions, 329 generalized Pareto distribution, 129, 130, 136 generation life table, see life table geometric distribution, 71-72, 94, 118-119, 132-135, 275, 343 Gini's test statistic, 440 Gnedenko test for exponentiality, see exponential distribution Gnedenko, Boris, 428 Gompertz distribution, 128-129, 130, 136 goodness-of-fit tests, 15-16, 415-432 Greenwood's formula, 384

## H

Haenszel, William, 385 Harris, Carl. 439 hazard function, 11-12, 63-67, 70 bathtub-shaped, 11, 65-67 derivation, 63-64 interpretation, 65-67 units. 65 heat pump compressor failure times, 400-402 highly accelerated life tests, 366 highly accelerated stress screens, 366 holding time, 212, 214 horse kick data, 417-419 hyperexponential distribution, 129-130, 136, 167, 182 hypergeometric distribution, 386 hypoexponential distribution, 129, 130, 136 hypothesis testing, 298-299 exercises, 333-338, 341, 343 exponential populations, 306-310, 352 power function, 308-310 significance level, 298

#### I

IDB distribution, 129, 130, 136, 154 IFR (increasing failure rate), 81-85, 106, 107, 149, 152 IFRA (increasing failure rate on the average), 85, 106, 107, 149 IFRA closure theorem, 85 improving item, 190 incomplete gamma function, 119, 451 independent increments, 191-192, 197-198 indicator variable, 171 inequality for vectors, 30 infinitesimal generator matrix, 211, 212, 214, 217, 218, 220 integrated circuit failure times, 412 intensity function, 63, 190 interaction term, 171, 184, 370-372 International Reliability Physics Symposium, 19 interval censoring, see censoring interval estimation, see confidence interval invariance property, see likelihood theory inverse gamma distribution, 276 inverse Gaussian distribution, 128, 130, 136, 252-253, 279, 329 irrelevant component, 25-26

# J

Jeffreys confidence interval, 378–379 jet engine failure times, 437 Johnson system of distributions, 137

## K

*k*-out-of-*n* system, 23–24, 38–39
Kaplan, Edward, 381
Kaplan–Meier product–limit estimator, 15 derivation, 380–384, 461–462 exercises, 409–411
Kolmogorov–Smirnov test, 15–16, 419–428 adjusted test statistic, 436 critical values, 423 exercises, 433–439 parameters estimated from data, 426–428 right-censored data sets, 423–426 test statistic, 419–423
kurtosis, 74, 86, 114, 121, 125, 133–135, 137

#### L

least squares estimators, 342 left censoring, *see* censoring

leukemia nonmaintained group survival times, 411 leukemia remission times, 285, 290-291, 294, 302-305, 328, 332, 333, 349-352, 363-364, 368, 369, 381-384, 386-388, 409-413 leukemia survival times, 372 life table, 15, 388–390 life testing, 307-312 exercises, 330-343 sample size determination, 307-310, 333, 334, 338 sequential, 312, 334 truncated, 311 with replacement, 311 lifetime distribution relationships, 130, 137 lifetime distribution representations, 60-72 cumulative hazard function, 67, 70 hazard function, 63-67, 70 mean residual life function, 67-68, 70-71 probability density function, 62-63 probability mass function, 70 relationships between, 68 survivor function, 61-62, 70 likelihood theory, 247-253 asymptotic properties, 254-257 censoring, 263-265 Fisher information matrix, 248-253 geometry, 248 invariance property, 251, 275, 293 likelihood ratio statistic, 254, 303, 315 maximum likelihood estimator, 248-253, 272-282, 330-342 observed information matrix, 252-253 score vector, 248-253 limiting availability, 206-211 limiting average availability, 206-211 link function, 170 load-sharing parallel system, see shared-parallel system location parameters, 108-109 location-scale family of distributions, 154 log logistic distribution, 128, 130, 132-136, 148-152, 169, 172, 279, 341, 368, 370 log logistic regression model, 183 log normal distribution, 128, 130, 132-136, 149, 152-154, 272, 341 log-linear link function, 170, 344 log-location-scale family, 329

log-rank test, 385–388 exercises, 411, 412 logistic distribution, 136, 150 logistic regression model, 373 logistic-exponential distribution, 136, 152 lung cancer survival times, 368-369, 371, 411

#### Μ

Makeham distribution, 129, 130, 136 Mantel, Nathan, 385 Markov models, 211-222 birth-death model, 218-222 continuous-time Markov chain, 211 holding time, 212, 214 infinitesimal generator matrix, 211, 212, 214, multivariate exponential distribution, 137 217, 218, 220 state space, 211 transition diagram, 208, 212, 213, 217-219 Markov's inequality, 86 maximal scale invariant statistic, 117 maximum likelihood estimation, see likelihood theory McDonald inverted beta distribution, 136 mean, 73, 114, 121, 125, 146 mean residual life function, 67-68, 70-71, 88-95 mean squared error, 239 median, 74, 146, 332 median rank regression, 329 Meier, Paul, 381 Mellin transform, 61, 85, 90 memoryless property, 94, 111–112, 217, 222, 441 nonparametric methods, 15, 374–414 method of moments, 274 Mill's ratio. 63. 128 minimal cut set, 29-34, 46 definition. 31 minimal cut vector, 31 minimal path set, 29-34, 46 definition, 30 minimal path vector, 30 minimal repair, 199-204, 226, 231 minimum wage data set, 325–327 minimum-variance unbiased estimator, 237-238 mixed discrete-continuous distribution, 72, 86 mixtures, 83-84, 165-169, 176 continuous mixtures, 168-169 exercises, 180-182 finite mixtures, 166-168, 325-327

mode, 73, 146 model adequacy, 15-16, 415-440 modular decomposition, 46 moment generating function, 61, 85 moment ratio diagrams, 130-135, 137, 154 moments, 72-75, 114, 121 Monte Carlo simulation, 113-114, 161, 202, 220-222, 225-228, 231-232, 243, 247, 271-273, 277-278, 332, 335, 340, 410, 433 MTBF (mean time between failures), 73 MTTF (mean time to failure), 73, 209, 335 MTTR (mean time to repair), 209 multiple decrements, see competing risks multistate system, 41 Muth distribution, 127, 130, 132-136, 283

#### Ν

NBU (new better than used), 86, 106, 107 NBUE (new better than used in expectation), 86, 107, 149 Nelson-Aalen estimator, 407 net lifetimes, 157-165, 177-180 Newton-Raphson procedure, 313, 314, 328, 455 non-central chi-square distribution, 419 nonhomogeneous Poisson process, 197-204 definition. 197 exercises, 224-228, 342, 413, 414 nonparametric estimation, 392-402 overlapping realizations, 398-402 power law process, 198, 224, 225, 227, 342 nonrepairable systems grouped data, 388-390 raw data, 374-388 repairable systems grouped data, 402-406 raw data, 390-402 normal distribution, 129, 132-135, 254-257

## 0

O-rings, 5-7 observed information matrix, 252-253 accelerated life model, 346-347, 367 exponential( $\lambda$ ) population, 289, 295, 333 exponential( $\theta$ ) population, 332 proportional hazards model, 362 odds ratio, 183

Operation Eagle Claw, 210 order statistics, 97, *115–117*, 266, 337 Ornstein–Uhlenbeck process, 136 overdispersed renewal process, 193–194

## Р

parallel system, 10, 23, 36-38, 43, 216 exponential component lifetimes, 77-78 parameters, 108-110, 126, 135 location, 108-109 scale, 108-109 shape, 108-109 Pareto distribution, 90, 91, 128, 130, 136, 149, 224.275 Pasztor, Andy, 8 path vector, 30 path vector technique for calculating system reliability, 39 Pearson system of distributions, 137 perfect repair, 199-204 Peto, Julian, 388 phase-type distributions, 137, 176, 266 piecewise exponential distribution, 136 point availability, 206-211 point estimation, 233-239 consistency, 239, 376, 394 Cramér-Rao inequality, 237-238, 266, 269 mean squared error, 239 minimum-variance unbiased estimator, 237-238 relative efficiency, 236-237, 337 unbiased estimator, 234-237, 376 point processes, 189-204 minimal repair, 199-204 392-402 perfect repair, 199-204 Poisson process, 192-193 renewal process, 193-197 superpositioning, 204, 392, 393 Poisson distribution, 94, 96, 117, 132-135, 169, 192, 195, 227, 229, 273, 278, 394 Poisson process, 117, 192–193, 312 power distribution, 153 power law process, 198, 224, 225, 227, 342 power Muth distribution, 136 power-loglinear intensity function, 227 prediction interval, 272

preventive maintenance, 188 prior distribution, 135 probability density function, 62-63 probability integral transformation, 113, 136 probability mass function, 70 probability plot, 291-293, 318-325 producer's risk, 307-308 product-limit estimator, see Kaplan-Meier estimator progressive censoring, see censoring proportional hazards model, 172-175, 356-366 baseline known, 356-357 baseline unknown, 357-366 exercises, 184-185, 370-373 inference, 356-366 matrix formulation, 344-345 references, 177, 367 right censoring, 361-363 tied observations, 363 proportional intensity model, 223

## Q

quality control, 1, *4–5* quartiles, 74

## R

random censoring, see censoring random sample, 234 random variate generation, 93, 106, 113-114 rank vector, 358-363 rat survival times, 271, 277, 279 rate function, 63 rate of occurence of failures (ROCOF), see intensity function nonhomogeneous Poisson process, 197-204, Rayleigh distribution, 119, 132-135, 263-265, 270, 273, 282, 341 Reagan, Ronald, 92 redistribute-to-the right algorithm, 382-384 redundancy, 5 relative efficiency, 236-237, 337 reliability bounds, 43-44, 47 calculation of system reliability, 35-43 cut vector technique, 39-40 decomposition technique, 40-41 definition of r(p), 35–36 expected value of  $\phi(\mathbf{X})$ , 36–39 path vector technique, 39

definition, 2–5 environmental conditions, 4 function, 34-43 growth, 7-10, 87 importance, 41-43 time scale, 3-4 versus quality, 4-5 Reliability & Maintainability 2000 initiative, 210- shake and bake, 172 211 Reliability and Maintainability Symposium, 19 reliability block diagram, see block diagram reliability bounds, 43-44, 47 reliability demonstration test, 312, 329 reliability function, 34-43 reliability importance, 41-43 reliability-centered maintenance, 222 renewal process, 193-197 overdispersed, 193-194 renewal equation, 196-197 underdispersed, 193-194 repairable systems, 12-13, 186-232 age replacement policy, 187-188 alternating renewal process, 205-211 block replacement policy, 187-188 definition, 187 failure replacement policy, 187-188 maintenance models, 188 minimal repair, 199-204 perfect repair, 199-204 point processes, 189-204 repair models, 189 replacement models, 187-188 reversed failure rate, 61 reversed hazard rate, 85 right censoring, see censoring risk set, 357-363

#### S

sample kurtosis, 135 sample mean, 135 sample size, 258 sample size determination, *see* life testing sample skewness, 135 sample standard deviation, 135 sampling distribution, 234 scale parameters, 108–109 scaled intensity function, 226 Schoenfeld residuals, 367, 440 Schoenfeld, David, 440 score vector, 248-253 self-reproducing property, 115, 123, 443 sequential k-out-of-n system, 46 series system, 5, 10, 22-23, 35-36, 42, 202-204, 213, 219 exponential component lifetimes, 76-77, 118 shape parameters, 108-109 shared-parallel system, 37, 140 signature, see system signature significant digits, 241 skewness, 74, 114, 121, 125, 132-135 snubber failure times, 410 socket model, see standby system Space Shuttle failure times, 281 standard deviation, 73 standard Wald distribution, 274 standby system, 37-38, 104, 117-118, 141, 148, 181, 192, 199, 202-205, 219 state space, 211 stationarity, 192 step-function estimator, 393 stochastic ordering, 87, 141 stochastic parameter models, see mixtures stress-strength model, 58, 329 stripchart, 393 structural importance, 27-29, 46 structure function, 21-29 definition, 22 superpositioning, 204, 392, 393 survivor function, 11-12, 61-62, 70 conditional, 12, 62 confidence interval for, 376-385 empirical, 288, 375 system lifetime distributions, 75-81 system redundancy allocation, 55 system reliability bounds, 43-44, 47 system reliability calculation, see reliability system signature, 47, 107 system state vector, 22

#### Т

*t* distribution, 245 test for exponentiality, *see* exponential distribution three-parameter Weibull distribution, 329 time-to-event data, 258 *Titanic* sinking, 1, 20 total time on test, 287, 290, 295, 349 total time on test transform, 85, 91 transition diagram, 208, 212, 213, 217–219 tridiagonal matrix, 218 truncated data, 267, 329 truncated normal distribution, 137 turbine wheel failure times, 339 Type I right censoring, *see* censoring Type II right censoring, *see* censoring

#### U

U.S.S. Halfbeak engine failure times, 392–396, 403–406 unavailability, 210–211 unbiased estimator, 234–237, 376 unbiasing constant, 269, 277, 331 underdispersed renewal process, 193–194 undersea digital repeaters, *see* electronic device failure times uniform distribution, 79–81, 90, 127–128, 130– 135, 149, 152, 167–168 unit exponential distribution, *see* exponential distribution

#### V

variance, *73*, 114, 121, 125, 337 variance–covariance matrix, *249–250*, 254–257, 317, 351 variate generation, *see* random variate generation, Monte Carlo simulation

## W

Wald confidence interval, 271, 377 Wald distribution, 274 warranty period, 75, 87, 92, 94, 109 wear out, 189-190 Weibull distribution, 14-15, 119-123, 312-327 characteristic life, 120 distribution classes, 130 distribution representations, 119, 127 exercises, 145-147, 339-341 Fisher information matrix, 314 fractiles, 75, 123 hazard function, 64 in a renewal process, 194, 225 in a standby system, 202-204 in competing risks, 164-165 in the accelerated life model, 171-172

in the proportional hazards model, 173-174 maximum likelihood estimators, 312-314, 455-458 moment ratio diagrams, 132-135 moments, 121 observed information matrix, 314 parameterizations, 120-121 probability plot, 318-325 references, 136, 328-329 relationship to extreme value distribution, 123relationship to gamma distribution, 125 self-reproducing property, 123 statistical inference, 312-327 Wilson-score confidence interval, 378-379 wooden beam failures, 371

#### Ζ

zero failure life test, 279, 280, 413 zero-inflated Poisson distribution, 274 zero-truncated Poisson distribution, 274