The various common discrete distributions that were introduced in the previous chapter and the common continuous distributions that have been introduced in this chapter were presented in separate sections. Isolating the presentation of the probability distributions in this fashion is unfortunate because many of these distributions are related to one another through special cases or transformations. Still another way to relate these probability distributions to one another is to place them on a graph of their moments. These graphs are often known as *moment-ratio diagrams*. A moment-ratio diagram is the locus of pairs of standardized moments for a particular probability distribution plotted on a single set of axes. Moment-ratio diagrams are useful for (1) quantifying the "distance" or "proximity" between univariate probability distributions based on their second, third, and fourth moments, (2) illustrating the limiting behavior of probability distributions, (3) highlighting the versatility of a particular probability distribution based on the range of values that the moments can assume, and (4) generating a list of potential probability models based on a data set.

As one illustration of a moment-ratio diagram, Figure 5.15 contains a plot of the population skewness

$$\gamma_3 = E\left[\left(\frac{X-\mu_X}{\sigma_X}\right)^3\right]$$

on the vertical axis, versus the population coefficient of variation

$$\gamma_2 = \frac{\sigma_X}{\mu_X}$$

on the horizontal axis, where μ_X and σ_X are the population mean and the population standard deviation of the random variable X. Some features of this moment-ratio diagram are listed below.

- The locus of points associated with the various probability distribution consist of either a single point, for example, the Benford and Rayleigh distributions, a curve, for example, the geometric and Bernoulli distributions, or a region, for example, the beta distribution.
- There are two gathering points: the exponential distribution at $(\gamma_2, \gamma_3) = (1, 2)$ and a degenerate distribution at $(\gamma_2, \gamma_3) = (0, 0)$.
- The Poisson distribution, with $\gamma_3 = \gamma_2$, and the gamma distribution, with $\gamma_3 = 2\gamma_2$, have linear relationships between γ_2 and γ_3.
- The fact that the beta distribution covers the largest amount of territory in Figure 5.15 is further evidence of its versatility as a probability model. This confirms the flexibility that was displayed in the wide assortment of probability density function shapes in Figure 5.12. Although the support of the beta distribution, $\mathcal{A} = \{x \,|\, 0 < x < 1\}$, limits its applicability, multiplying a beta random variable by a positive constant makes it a reasonable survival model. A positive shift parameter can also be added to further enhance its applicability.
- The limiting values of the beta distribution region are the line associated with the gamma distribution $\gamma_3 = 2\gamma_2$ and the curve associated with the Bernoulli distribution $\gamma_3 = \gamma_2 - 1/\gamma_2$.
- Symmetric distributions, such as the $N\left(\mu, \sigma^2\right)$, $U(a, b)$, and discrete uniform distributions, all have population skewness $\gamma_3 = 0$.
- The curves associated with the gamma and Weibull distributions intersect at the exponential distribution, which is associated with shape parameters equal to $\kappa = 1$.
- The open point associated with the Pareto distribution parameterized as $f(x) = \kappa\lambda^\kappa / x^{\kappa+1}$ for $x > \lambda$ gives the limiting distribution as $\kappa \to \infty$. The values of γ_2 and γ_3 are defined for $\kappa > 3$.

- The chi-square distribution, indicated by a C for various values of its degrees of freedom, and the Erlang distribution, indicated by an E for various values of its integer shape parameter, coincide when the degrees of freedom for the chi-square distribution are even. This accounts for the alternating pattern of C and CE labels along the line for the gamma distribution.

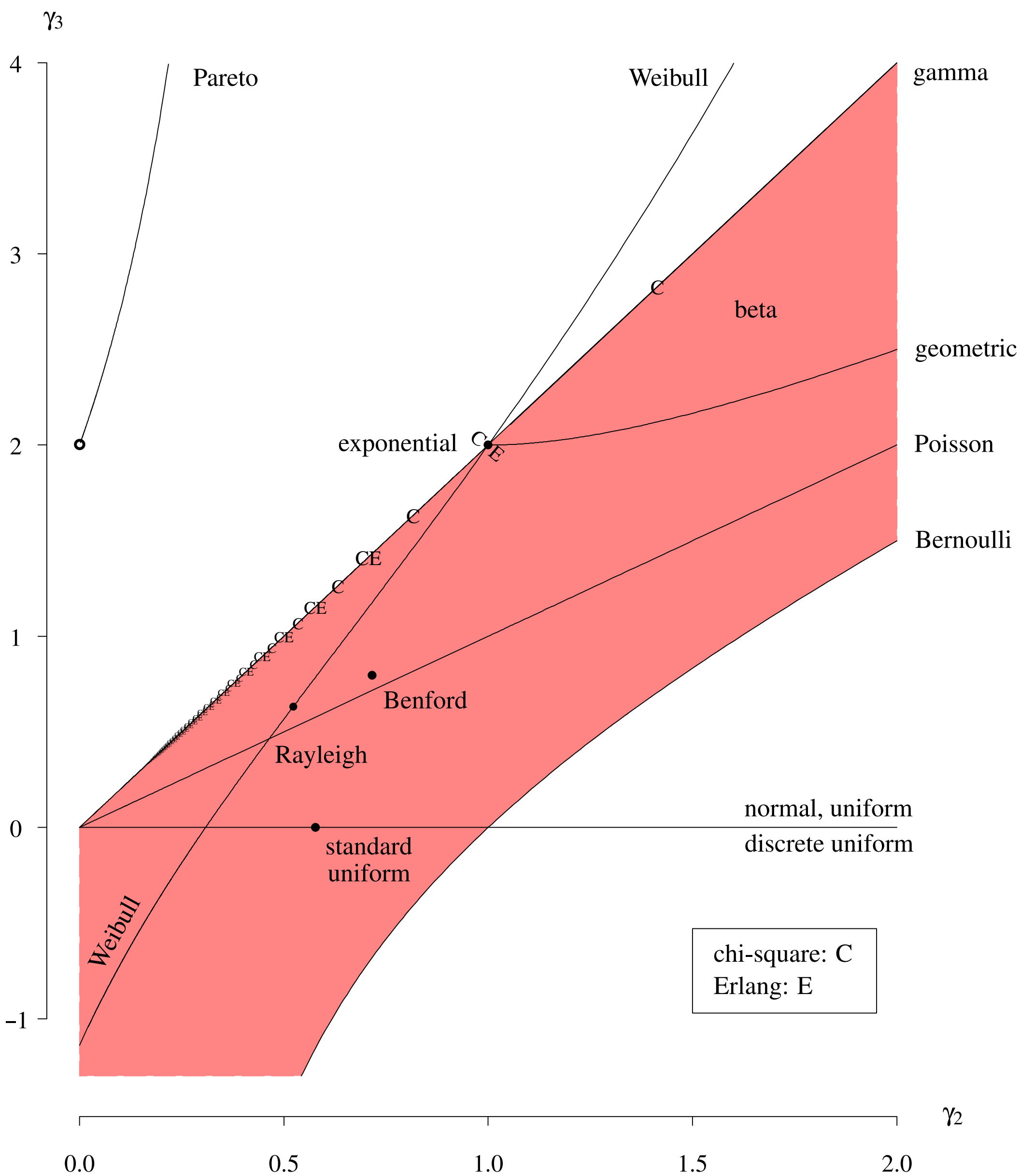

Figure 5.15: Population skewness γ_3 vesus population coefficient of variation γ_2.