This section closes with one final unifying example that stresses the importance of the following two questions associated with a counting problem: (a) Is the sampling performed *with replacement* or *without replacement*? (b) Is the sample considered *ordered* or *unordered*?

Example 1.42 How many ways are there to select 4 billiard balls from a bag containing the 15 balls numbered $1, 2, \ldots, 15$?

The question as stated is (deliberately) vague. It has not been specified whether

- the billiard balls are replaced (that is, returned to the bag) after being sampled, and
- the order that the balls are being drawn from the bag is important.

So there are really $2 \times 2 = 4$ different questions being asked here. The answers to these questions are given in the 2×2 matrix below.

	Without replacement	With replacement
Ordered sample	$15 \cdot 14 \cdot 13 \cdot 12$	$15 \cdot 15 \cdot 15 \cdot 15$
Unordered sample	$\binom{15}{4}$	$\binom{18}{4}$

These simplify to

	Without replacement	With replacement
Ordered sample	32,760	50,625
Unordered sample	1365	3060

There are several observations that can be made on the numbers in this 2×2 matrix. First of all, the entries in column 2 are always greater than the corresponding entries in column 1. This is because sampling with replacement allows for more possible draws due to the fact that the size of the population from which a draw is made remains constant rather than diminishing. Secondly, the entries in row 1 are always greater than the corresponding entries in row 2. This is because the count of ordered draws (permutations) will always exceed the corresponding number of unordered draws (combinations).

A further explanation of the lower-right entry of the matrix might be needed. Consider 15 bins and 4 balls, where $\bigcirc$ denotes a billiard ball. One draw of 4 balls is depicted below.

1	|	2	|	3	|	4	|	5	|	$\cdots$	|	14	|	15
	|	$\bigcirc\bigcirc$	|		|	$\bigcirc$	|		|	$\cdots$	|		|	$\bigcirc$

This arrangement of bins and markers corresponds to the unordered draw 2, 2, 4, 15 taken with replacement from the bag. We need to count the number of arrangements of 14 dividers plus 4 balls, or a total of 18 objects. Since the $\bigcirc$'s are indistinguishable, there are

$$\binom{18}{4}$$

different orderings.