4.8 Exercises

4.1 Let $X \sim \text{Bernoulli}(p)$. Find $E[\cos(\pi X)]$.

4.2 Let $X \sim \text{Bernoulli}(p)$. Find the value of p that maximizes $V[X]$.

4.3 A binomial random variable X has population mean 3 and population variance 2. Write an R statement that calculates $P(X \le 2)$.

4.4 A certain stock is priced daily in integer dollar values and begins at \$100. Every day the value of this stock goes up one dollar with probability p or goes down by one dollar with probability $1-p$, for some probability p satisfying $0 < p < 1$. Scott places \$100 in an investment known as a "structured note" which gives him one of two outcomes:

- If the price of the stock at the end of 30 trading days is greater than or equal to \$80, Scott earns 7% on the investment, regardless of the value of the stock, that is, he receives \$107.
- If the price of the stock at the end of 30 trading days is less than \$80, Scott gets the value of the stock at the end of 30 trading days.

Find p (accurate to four digits) such that the expected value of the structured note at the end of 30 trading days is \$103.

4.5 Ten fair dice are rolled. Let A_i denote the event that the number of spots on the up face of die i is four, for $i = 1, 2, \ldots, 10$. Find $P(A_1 \cup A_2 \cup \ldots \cup A_{10})$.

4.6 One version of the *PowerBall* lottery requires matching the numbers on five white balls, in any order, selected at random and without replacement from a drum containing 69 white balls numbered $1, 2, \ldots, 69$, and matching an additional red "powerball" number selected at random from a second drum containing 26 red balls numbered $1, 2, \ldots, 26$.

(a) What is the probability that one person who selects six numbers wins the *PowerBall* lottery?

(b) If one thousand people purchase *PowerBall* lottery tickets simultaneously and each person makes a random selection for the six numbers, independent of any of the other gamblers, find the probability mass function of the number of winners.

4.7 Let $X \sim \text{binomial}(3, p)$. Write an equation in p that satisfies $P(X = 0) = P(X \ge 2)$.

4.8 Kelsey begins on a number line at the origin. She has a biased coin which shows heads, when flipped, with probability p, for $0 < p < 1$. She flips the coin. If it shows heads, she moves one unit to the right on the number line to 1. If it shows tails, she moves one unit to the left on the number line to -1. She conducts this random experiment a total of n times, moving to the right or to the left from her current position one unit based on the outcome of the experiment. Find the probability mass function of her location on the number line after n flips.

4.9 The probability mass function for a binomial(n, p) random variable X is

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0, 1, 2, \ldots, n,$$

for some positive integer n and $0 < p < 1$. What is the probability mass function $f(x)$ in the limit as $p \to 0$ and $p \to 1$?