## 3.6 Exercises

**3.1** An urn contains $r$ red balls and $w$ white balls, where $r$ and $w$ are positive integers and $r \geq 3$. Balls are drawn successively and without replacement. Let the random variable $X$ be the draw number in which the third red ball is drawn. Find the probability mass function of $X$.

**3.2** Atlas and Bruce agree to engage in the following test of strength. They will have consecutive arm wrestling matches until one of them wins two matches in a row and is declared the winner. Atlas wins a given match with probability $3/5$. Assuming that the matches are independent, give the probability mass function of the number of matches required to declare a winner.

**3.3** Marian rolls five fair dice simultaneously. Find the probability that the total number of spots showing is less than or equal to 11.

**3.4** A fair die is rolled $n$ times. Let $X_i$ denote the number of spots that are on the up face on roll $i$, for $i = 1, 2, \ldots, n$. Find the probability mass function of $Y = \max\{X_1, X_2, \ldots, X_n\}$.

**3.5** The R function `bubblesort` is given below.

```
bubblesort = function(a) {
  n = length(a)
  for (i in 1:(n - 1)) {
    for (j in 1:(n - i)) {
      if (a[j + 1] < a[j]) {
        tmp = a[j]
        a[j] = a[j + 1]
        a[j + 1] = tmp
      }
    }
  }
return(a)
}
```

A *swap* occurs when the inside `if` statement is executed. If a vector containing a random permutation of three distinct integers is passed to `bubblesort`, give the probability mass function of the number of swaps required by the code to sort the random permutation.

**3.6** A fair green die and a fair red die are tossed together. Let $X$ denote the number of spots showing on the green die and $Y$ denote the number of spots showing on the red die. Find the probability mass function for:

(a) the sum of the spots showing on the two dice, $X + Y$,

(b) the number of spots showing on the green die minus the number of spots showing on the red die, $X - Y$,

(c) the difference between the spots showing on the two dice, $|X - Y|$,

(d) the maximum number of spots showing on a single die, $\max\{X, Y\}$,

(e) the minimum number of spots showing on a single die, $\min\{X, Y\}$.