3.6 Exercises

3.1 An urn contains \( r \) red balls and \( w \) white balls, where \( r \) and \( w \) are positive integers and \( r \geq 3 \). Balls are drawn successively and without replacement. Let the random variable \( X \) be the draw number in which the third red ball is drawn. Find the probability mass function of \( X \).

3.2 Atlas and Bruce agree to engage in the following test of strength. They will have consecutive arm wrestling matches until one of them wins two matches in a row and is declared the winner. Atlas wins a given match with probability \( \frac{3}{5} \). Assuming that the matches are independent, give the probability mass function of the number of matches required to declare a winner.

3.3 Marian rolls five fair dice simultaneously. Find the probability that the total number of spots showing is less than or equal to 11.

3.4 A fair die is rolled \( n \) times. Let \( X_i \) denote the number of spots that are on the up face on roll \( i \), for \( i = 1, 2, \ldots, n \). Find the probability mass function of \( Y = \max\{X_1, X_2, \ldots, X_n\} \).

3.5 The R function \( \text{bubblesort} \) is given below.

```r
bubblesort = function(a) {
  n = length(a)
  for (i in 1:(n - 1)) {
    for (j in 1:(n - i)) {
      if (a[j + 1] < a[j]) {
        tmp = a[j]
        a[j] = a[j + 1]
        a[j + 1] = tmp
      }
    }
  }
  return(a)
}
```

A \textit{swap} occurs when the inside if statement is executed. If a vector containing a random permutation of three distinct integers is passed to \( \text{bubblesort} \), give the probability mass function of the number of swaps required by the code to sort the random permutation.

3.6 A fair green die and a fair red die are tossed together. Let \( X \) denote the number of spots showing on the green die and \( Y \) denote the number of spots showing on the red die. Find the probability mass function for:

(a) the sum of the spots showing on the two dice, \( X + Y \),
(b) the number of spots showing on the green die minus the number of spots showing on the red die, \( X - Y \),
(c) the difference between the spots showing on the two dice, \( |X - Y| \),
(d) the maximum number of spots showing on a single die, \( \max\{X, Y\} \),
(e) the minimum number of spots showing on a single die, \( \min\{X, Y\} \).