So far, Monte Carlo simulation has only been used to support known analytic results. It is also possible to use Monte Carlo simulation on problems where the exact solution is not known.

Example 3.22 What is the 90th percentile of the distance between two points chosen at random in the interior of a unit square?

The analytic tools presently available do not allow us to solve this problem analytically, although it poses no difficulty for Monte Carlo simulation. One way to interpret the phrase "two points chosen at random" from the problem statement is to assume that the coordinates of the two points in the unit square, (x_1, y_1) and (x_2, y_2), are generated using four random variates that are uniformly distributed between 0 and 1. Using the distance formula, the distance between the two points,

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2},$$

can range from 0 (when the points are identical) to $\sqrt{2}$ (when the points are at opposite corners of the square). Three realizations of random line segments in the unit square are shown in Figure 3.25.

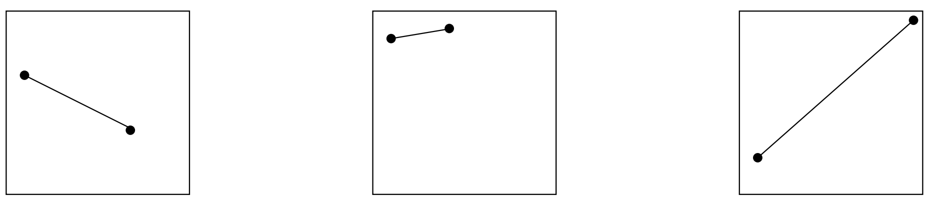

Figure 3.25: Three realizations of random line segments in a unit square.

The R code to estimate the 90th percentile of the distance between the two points is given below.

```
nrep = 1000000
x1 = runif(nrep)
x2 = runif(nrep)
y1 = runif(nrep)
y2 = runif(nrep)
d  = sqrt((x1 - x2) ^ 2 + (y1 - y2) ^ 2)
sort(d)[0.9 * nrep]
```

Of the 1,000,000 distances generated and stored in the vector `d`, the 900,000th ordered distance is the Monte Carlo estimate for the 90th percentile. After a call to `set.seed(3)` to initialize the random number stream, five runs of the Monte Carlo simulation code are run, yielding

0.8592064 0.8585644 0.8589713 0.8582711 0.8587005.

How many digits should be reported? Since four of the five results round to 0.859, it is reasonable to report the estimate of the 90th percentile of the distance between the points as

$$x_{0.9} \cong 0.859.$$