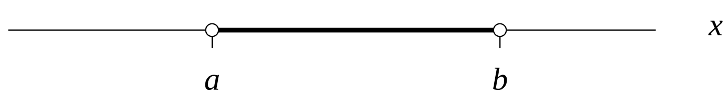
- Inequalities
  - Inequalities indicate the ordering of quantities on a number line
  - The expression $2 < 5$ is read "two is less than five" and indicates that 2 lies to the left of 5 on a number line
  - Inequalities are mathematical statements of the form

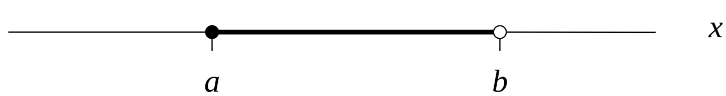
$x < y$	x is less than y
$x \leq y$	x is less than or equal to y
$x > y$	x is greater than y
$x \geq y$	x is greater than or equal to y

  - The inequalities $x < y$ and $x > y$ are known as *strict inequalities*
  - Interval notation
    * The set of x values satisfying $a \leq x \leq b$ is a *closed interval* and is denoted by $[a, b]$

    * The set of x values satisfying $a < x < b$ is an *open interval* and is denoted by (a, b)

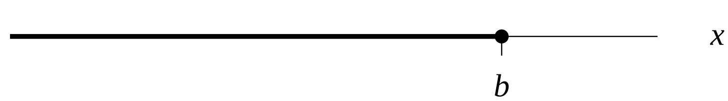


    * The set of x values satisfying $a \leq x < b$ is denoted by $[a, b)$



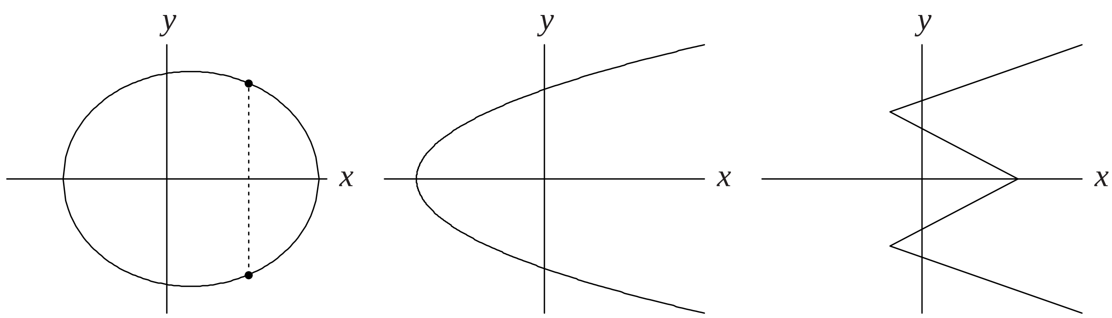
    * The set of x values satisfying $a < x \leq b$ is denoted by $(a, b]$

    * The set of x values satisfying $x \leq b$ is denoted by $(-\infty, b]$

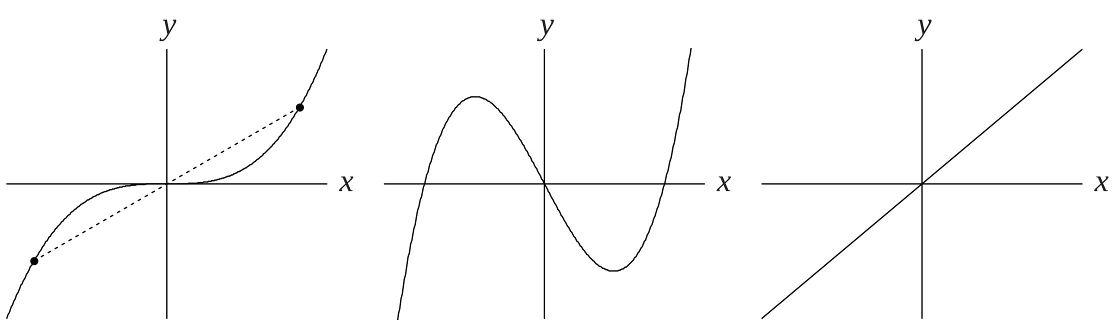


    * The set of x values satisfying $-\infty < x < \infty$ is the entire real number line and is denoted by $(-\infty, \infty)$
  - For a positive constant a, $|x| = a$ is equivalent to $x = \pm a$
  - For a positive constant a, $|x| < a$ is equivalent to $-a < x < a$

– The graph of an equation is *symmetric about the x-axis* if replacing y with $-y$ in the equation results in the original equation, that is, if (x, y) is a point on the graph of the equation, then $(x, -y)$ is also on the graph. Geometrically, every point on the graph has a mirror image on the opposite side of the x-axis. Three examples are shown below.

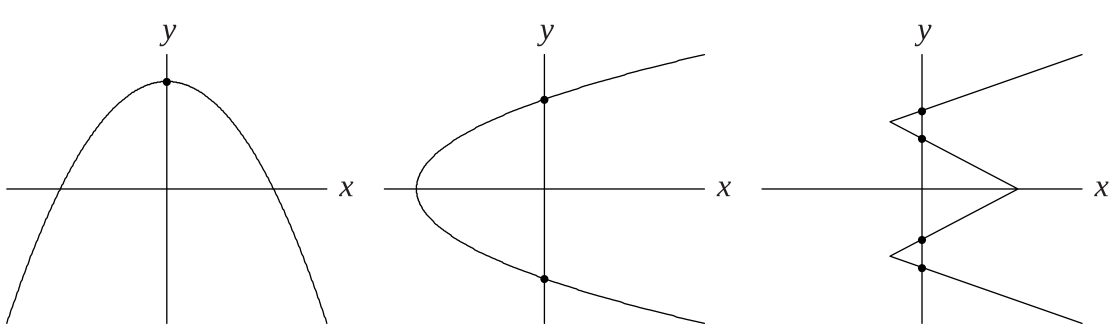


– The graph of an equation is *symmetric about the origin* if replacing x with $-x$ and y with $-y$ in the equation results in the original equation, that is, if (x, y) is a point on the graph of the equation, then $(-x, -y)$ is also on the graph. Geometrically, every point on the graph has a mirror image on the opposite side of the origin. Three examples are shown below.

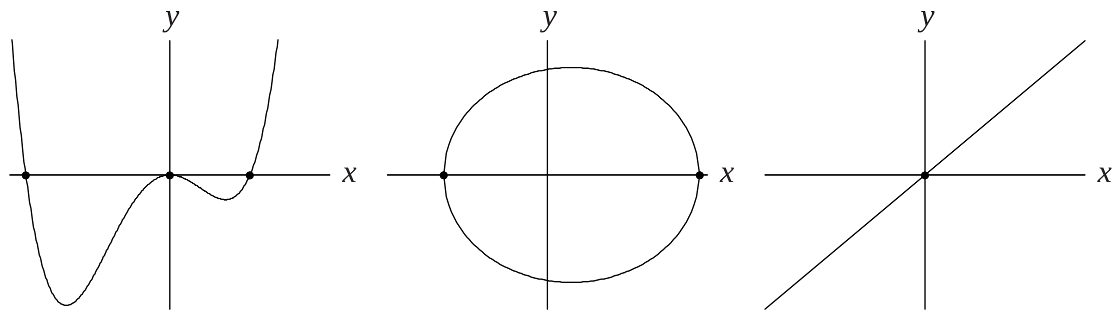


- Intercepts

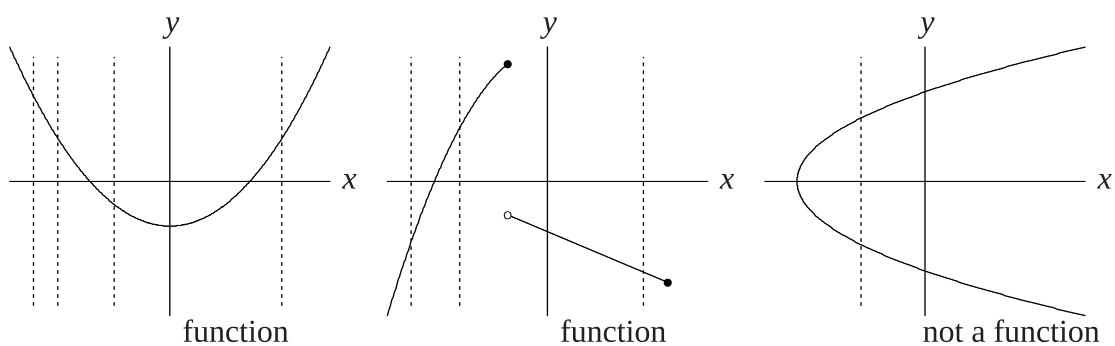
  – A *y-intercept* in the graph of an equation in x and y is a point where the graph touches or crosses the y-axis. The y-intercepts of a graph are found by setting x equal to zero and solving the resulting equation for y. Graphs with one, two, and four y-intercept(s) are shown below.



- An *x-intercept* in the graph of an equation in x and y is a point where the graph touches or crosses the x-axis. The x-intercepts of a graph are found by setting y equal to zero and solving the resulting equation for x. Graphs with three, two, and one x-intercept(s) are shown below.



- Functions
  - A *function* $y = f(x)$ pairs each element in the *domain* (the set of values allowed for x) with exactly one element in the *range* (the set of values allowed for y)
  - Functional notation
    * f is the name of the function
    * x is the *independent* variable
    * y is the *dependent* variable
    * $f(x)$ is the value of the function associated with x, read as "f of x"
    * other letters can be substituted for x, y, and f
  - *Vertical line test*: A graph is associated with a *function* if a vertical line drawn at any x position intersects the graph at most once. If one or more vertical lines can be drawn that intersect the graph two or more times, then the graph does not correspond to a function.



  - Examples of relationships between x and y that are not functions:
    * $x = y^2$
    * $x = |y|$
    * $x^2 + y^2 = 1$

4.30 The graph of the piecewise linear function $y = f(x)$ is shown on the left and the graph of the piecewise linear function $y = g(x)$ is shown on the right.

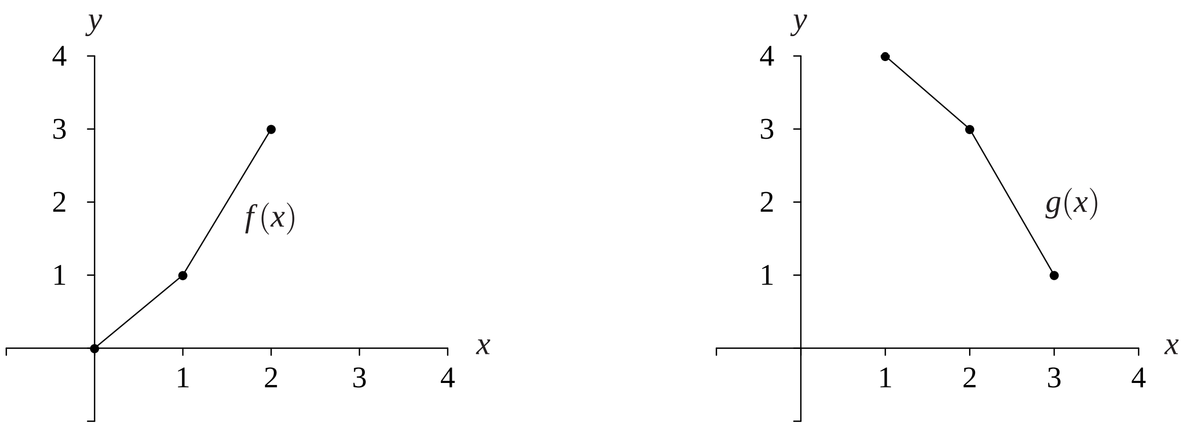
(a) Find $f(2) + g(1)$.

(b) Find $g\big(f(2)\big)$.

(c) Find $f^{-1}\big(g(2)\big)$.

(d) Find the domain of $f(x)/g(x)$.

(e) Using three rigid transformations, write $g(x)$ in terms of f and x.



4.30 The graph of the piecewise linear function $y = f(x)$ is shown on the left and the graph of the piecewise linear function $y = g(x)$ is shown on the right.

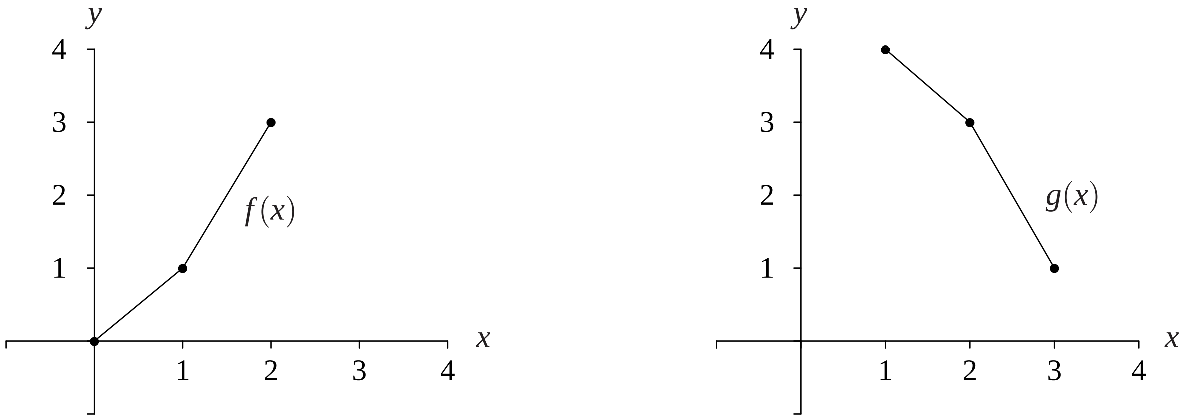
(a) Find $f(2)+g(1)$.

(b) Find $g\big(f(2)\big)$.

(c) Find $f^{-1}\big(g(2)\big)$.

(d) Find the domain of $f(x)/g(x)$.

(e) Using three rigid transformations, write $g(x)$ in terms of f and x.



(a) The sum of $f(2)$ and $g(1)$ is

$$f(2)+g(1) = 3+4 = 7.$$

(b) The composition of g and f evaluated at $x = 2$ is

$$g\big(f(2)\big) = g(3) = 1.$$

(c) The composition of f^{-1} and g evaluated at $x = 2$ is

$$f^{-1}\big(g(2)\big) = f^{-1}(3) = 2.$$

(d) The domain of the function $f(x)/g(x)$ is the intersection of the domain of $f(x)$, which is $\{x\,|\,x \in \mathbb{R}, 0 \le x \le 2\}$, and the domain of $g(x)$, which is $\{x\,|\,x \in \mathbb{R},\ 1 \le x \le 3\}$. The domain of $f(x)/g(x)$ is

$$\{x\,|\,x \in \mathbb{R},\ 1 \le x \le 2\}.$$

(e) In order to write $g(x)$ in terms of f and x, use the following rigid transformations (in order):

- shift the graph of $f(x)$ one unit to the right,
- rotate (reflect) this graph about the x-axis,
- shift this graph up four units.

Thus, $g(x)$ can be written in terms of $f(x)$ and x as

$$g(x) = -f(x-1)+4.$$

Index