Theorem The Weibull distribution has the variate generation property. That is, the inverse cumulative distribution function of a Weibull(α, β) random variable can be expressed in closed-form.

Proof The probability density function of a Weibull(α, β) random variable is

$$f(x) = (\beta/\alpha)x^{\beta-1}e^{-(1/\alpha)x^\beta} \qquad\qquad x > 0.$$

The cumulative distribution function is

$$F(x) = 1 - e^{-(1/\alpha)x^\beta} \qquad\qquad x > 0.$$

Equating the cumulative distribution function to u where $0 < u < 1$ yields the inverse cumulative distribution function

$$F^{-1}(x) = (-\alpha \ln(1-u))^{1/\beta} \qquad\qquad 0 < u < 1.$$

So a closed-form variate generation algorithm for the Weibull distribution is

generate $U \sim U(0, 1)$
$X \leftarrow (-\alpha \ln(1-u))^{1/\beta}$
return(X)

APPL verification: The APPL statements

```
X := WeibullRV(alpha, beta);
CDF(X);
IDF(X);
```

confirm the result.