Theorem The Weibull distribution has the scaling property. That is, If $X \sim \text{Weibull}(\alpha, \beta)$ then $Y = kX$ also has the Weibull distribution.

Proof Let the random variable X have the Weibull(α, β) distribution with probability density function

$$f(x) = \frac{\beta}{\alpha} x^{\beta-1} e^{-(1/\alpha)x^{\beta}} \qquad\qquad x > 0.$$

Let k be a positive, real constant. The transformation $Y = g(X) = kX$ is a 1–1 transformation from $\mathcal{X} = \{x \,|\, x > 0\}$ to $\mathcal{Y} = \{y \,|\, y > 0\}$ with inverse $X = g^{-1}(Y) = Y/k$ and Jacobian

$$\frac{dX}{dY} = \frac{1}{k}.$$

Therefore, by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| \\ &= \frac{\beta}{\alpha} (y/k)^{\beta-1} e^{-(1/\alpha)(y/k)^{\beta}} \, |1/k| \\ &= \frac{\beta}{\alpha k^{\beta}} \, y^{\beta-1} \, e^{(-1/\alpha k^{\beta}) y^{\beta}} \qquad\qquad y > 0, \end{aligned}$$

which is the probability density function of a Weibull$(\alpha k^{\beta}, \beta)$ random variable.

APPL verification: The APPL statements

```
assume(k > 0);
assume(alpha > 0);
assume(beta > 0);
X := [[x -> beta / alpha * x ^ (beta - 1) * exp(-x ^ beta / alpha)],
       [0, infinity], ["Continuous", "PDF"]];
g := [[x -> k * x], [0, infinity]];
Y := Transform(X, g);
```

confirm the result.