Theorem If $X_i \sim \text{Weibull}(\alpha_i, \beta)$, for $i = 1, 2, \ldots, n$, and $X_1, X_2, \ldots, X_n$ are mutually independent random variables, then

$$\min\{X_1, X_2, \ldots, X_n\} \sim \text{Weibull}\left(\left[\sum_{i=1}^{n}(1/\alpha_i)\right]^{-1}, \beta\right).$$

Proof The random variable X_i has probability density function

$$f_{X_i}(x) = (\beta/\alpha_i)x^{\beta-1}e^{-x^\beta/\alpha_i} \qquad x > 0,$$

for $i = 1, 2, \ldots, n$. The associated survivor function of X_i is

$$S_{X_i}(x) = e^{-x^\beta/\alpha_i} \qquad x > 0$$

for $i = 1, 2, \ldots, n$. Let $Y = \min\{X_1, X_2, \ldots, X_n\}$. The survivor function of Y is

$$\begin{aligned}
S_Y(y) &= P(Y \geq y) \\
&= P(\min\{X_1, X_2, \ldots, X_n\} \geq y) \\
&= P(X_1 \geq y, X_2 \geq y, \ldots, X_n \geq y) \\
&= P(X_1 \geq y)\, P(X_2 \geq y) \ldots P(X_n \geq y) \\
&= S_{X_1}(y) S_{X_2}(y) \ldots S_{X_n}(y) \\
&= e^{-(1/\alpha_1)x^\beta} e^{-(1/\alpha_2)x^\beta} \ldots e^{-(1/\alpha_n)x^\beta} \\
&= e^{-\sum_{i=1}^{n}(1/\alpha_i)x^\beta} \qquad x > 0.
\end{aligned}$$

This survivor function can be recognized as that of a Weibull random variable with scale parameter $\left[\sum_{i=1}^{n}(1/\alpha_i)\right]^{-1}$ and shape parameter β.

APPL illustration: The APPL statements to find the minimum of a $\text{Weibull}(2, \beta)$ and $\text{Weibull}(3, \beta)$ are (note different parameterizations between APPL and the chart):

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assume(beta > 0);
X1 := [[x -> exp(-x ^ beta / 2)], [0, infinity], ["Continuous", "SF"]];
X2 := [[x -> exp(-x ^ beta / 3)], [0, infinity], ["Continuous", "SF"]];
Minimum(X1, X2);
```

These statements yield a Weibull distribution for the minimum with a scale parameter

$$\left[\sum_{i=1}^{n}(1/\alpha_i)\right]^{-1} = \left[\frac{1}{2} + \frac{1}{3}\right]^{-1} = \frac{6}{5}$$

and shape parameter β, which is consistent with the theorem.