Theorem The natural logarithm of a Weibull($\alpha$, $\beta$) random variable is an extreme value ($\alpha$, $\beta$) random variable.

Proof Let the random variable $X$ have the Weibull distribution with probability density function

$$f_X(x) = \frac{\beta}{\alpha} x^{\beta-1} e^{-x^{\beta}/\alpha} \quad x > 0.$$  

The transformation $Y = g(X) = \log X$ is a 1–1 transformation from $\mathcal{X} = \{x \mid x > 0\}$ to $\mathcal{Y} = \{y \mid -\infty > y > \infty\}$ with inverse $X = g^{-1}(Y) = e^Y$ and Jacobian

$$\frac{dX}{dY} = e^Y.$$  

Therefore by the transformation technique, the probability density function of $Y$ is

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| = \frac{\beta}{\alpha} (e^y)^{\beta-1} e^{-e^y^{\beta}/\alpha} |e^y| = \frac{\beta}{\alpha} e^{\beta y - (e^y)^{\beta}/\alpha} \quad -\infty < y < \infty,$$

which is the probability density function of the extreme value distribution.

APPL verification: The APPL statements

```
assume(alpha > 0);
X := WeibullRV(((1 / alpha) ^ (1 / beta)), beta);
g := [[x -> log(x)], [0, infinity]];
Y := Transform(X, g);
Z := ExtremeValueRV(alpha, beta);
```

yield identical functional forms

$$f_Y(y) = \frac{\beta}{\alpha} e^{\beta y - (e^y)^{\beta}/\alpha} \quad -\infty < y < \infty,$$

for the random variables $Y$ and $Z$, which verifies that the natural logarithm of a Weibull random variable has the extreme value distribution. Notice that the first Weibull parameter is entered $(1/\alpha)^{(1/\beta)}$ so the parameterization will match that of the transformation technique above.