Theorem The natural logarithm of a Weibull($\alpha,\ \beta$) random variable is an extreme value ($\alpha,\ \beta$) random variable.

Proof Let the random variable X have the Weibull distribution with probability density function

$$f_X(x) = (\beta/\alpha)x^{\beta-1}\, e^{-x^\beta/\alpha} \qquad\qquad x > 0.$$

The transformation $Y = g(X) = \log X$ is a 1–1 transformation from $\mathcal{X} = \{x\,|\,x > 0\}$ to $\mathcal{Y} = \{y\,|\,-\infty > y > \infty\}$ with inverse $X = g^{-1}(Y) = e^Y$ and Jacobian

$$\frac{dX}{dY} = e^Y.$$

Therefore by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y))\left|\frac{dx}{dy}\right| \\ &= (\beta/\alpha)(e^y)^{\beta-1}\, e^{-(e^y)^\beta/\alpha}|e^y| \\ &= (\beta/\alpha)e^{\beta y-(e^y)^\beta/\alpha} \qquad\qquad -\infty < y < \infty, \end{aligned}$$

which is the probability density function of the extreme value distribution.

APPL verification: The APPL statements

```
assume(alpha > 0);
X := WeibullRV(((1 / alpha) ^ (1 / beta)), beta);
g := [[x -> log(x)], [0, infinity]];
Y := Transform(X, g);
Z := ExtremeValueRV(alpha, beta);
```

yield identical functional forms

$$f_Y(y) = (\beta/\alpha)e^{\beta y-(e^y)^\beta/\alpha} \qquad\qquad -\infty < y < \infty,$$

for the random variables Y and Z, which verifies that the natural logarithm of a Weibull random variable has the extreme value distribution. Notice that the first Weibull parameter is entered $(1/\alpha)^{(1/\beta)}$ so the parameterization will match that of the transformation technique above.