

Theorem The uniform distribution is a special case of the von Mises distribution when $\kappa = 0$.

Proof The von Mises distribution has probability density function

$$f(x) = \frac{e^{\kappa \cos(x-\mu)}}{2\pi I_0(\kappa)} \quad 0 < x < 2\pi,$$

where

$$I_0(\kappa) = \sum_{i=0}^{\infty} \frac{\kappa^{2i}}{2^{2i} (i!)^2}.$$

When $\kappa = 0$ this reduces to

$$f(x) = \frac{1}{2\pi} \quad 0 < x < 2\pi,$$

which is the probability density function of the $U(0, 2\pi)$ distribution.

Maple verification: The Maple statements

```
VM := exp(kappa * cos(x - mu)) / (2 * pi * sum((kappa ^ (2 * i)) /  
      (2 ^ (2 * i) * (i!) ^ 2), i = 0 .. infinity));  
eval(subs(kappa = 0, VM));
```

yield

$$f(x) = \frac{1}{2\pi} \quad 0 < x < 2\pi,$$

which is the probability density function of the $U(0, 2\pi)$ distribution.