

**Theorem** [UNDER CONSTRUCTION! The von Mises distribution appears to not have the scaling property.] The von Mises distribution has the scaling property. That is, if  $X \sim \text{von Mises}(\kappa, \mu)$  then  $Y = cX$  also has the von Mises distribution.

**Proof** [UNDER CONSTRUCTION! The von Mises distribution appears to not have the scaling property.] Let the random variable  $X$  have the von Mises( $\kappa, \mu$ ) distribution with probability density function

$$f(x) = \frac{e^{\kappa \cos(x-\mu)}}{2\pi I_0(\kappa)} \quad 0 < x < 2\pi,$$

where

$$I_0(\kappa) = \sum_{i=0}^{\infty} \frac{\kappa^{2i}}{2^{2i}(i!)^2}.$$

Let  $c$  be a positive, real constant. The transformation  $Y = g(X) = cX$  is a 1-1 transformation from  $\mathcal{X} = \{x \mid 0 < x < 2\pi\}$  to  $\mathcal{Y} = \{y \mid 0 < y < 2\pi\}$  with inverse  $X = g^{-1}(Y) = Y/c$  and Jacobian

$$\frac{dX}{dY} = \frac{1}{c}.$$

Therefore, by the transformation technique, the probability density function of  $Y$  is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| \\ &= \frac{e^{\kappa \cos(y/c-\mu)}}{2\pi I_0(\kappa)} \left| \frac{1}{c} \right|, \end{aligned}$$

which is the probability density function of a von Mises() random variable.