Theorem Random variates from the $U(a, b)$ distribution can be generated in closed-form by inversion.

Proof The uniform distribution has probability density function

$$f(x) = \frac{1}{b-a} \qquad\qquad a < x < b,$$

and cumulative distribution function

$$F(x) = \frac{x-a}{b-a} \qquad\qquad a < x < b.$$

Equating the cumulative distribution function to u, where $0 < u < 1$ yields an inverse cumulative distribution function

$$F^{-1}(u) = a + (b-a)u \qquad\qquad 0 < u < 1.$$

So a closed-form variate generation algorithm using inversion for the $U(a, b)$ distribution is

generate $U \sim U(0, 1)$
$X \leftarrow a + (b-a)U$
return(X)

APPL verification: The APPL statement

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IDF(UniformRV(a, b))
```

produced the inverse distribution function

$$F^{-1}(u) = a + (b-a)u \qquad\qquad 0 < u < 1.$$