Theorem The $U(a, b)$ distribution has the residual property, that is, the distribution left-truncated at some real constant c, where $a < c < b$, is also in the uniform family.

Proof The $U(a, b)$ distribution has probability density function

$$f(x) = \frac{1}{b-a} \qquad\qquad a < x < b$$

and associated survivor function

$$S(x) = \int_x^b f(t)dt = \int_x^b \frac{1}{b-a}dt = \left[\frac{t}{b-a}\right]_x^b = \frac{b-x}{b-a} \qquad\qquad a < x < b.$$

A $U(a, b)$ random variable that is truncated on the left at some real constant c, $a < c < b$, has survivor function

$$S_{X|X>c}(x) = \frac{S(x)}{S(c)} = \frac{\frac{b-x}{b-a}}{\frac{b-c}{b-a}} = \frac{b-x}{b-c} \qquad\qquad c < x < b.$$

The associated probability density function is

$$f_{X|X>k}(x) = \frac{1}{b-c} \qquad\qquad c < x < b$$

which is in the uniform family, that is, $X|X > c \sim U(c, b)$.

APPL verification: The APPL statements

```
X := UniformRV(a, b);
SF(X);
assume(c > a);
additionally(c < b);
SF(X)[1][1](x) / SF(X)[1][1](c);
```

verify that the conditional survivor function is also from the uniform family.