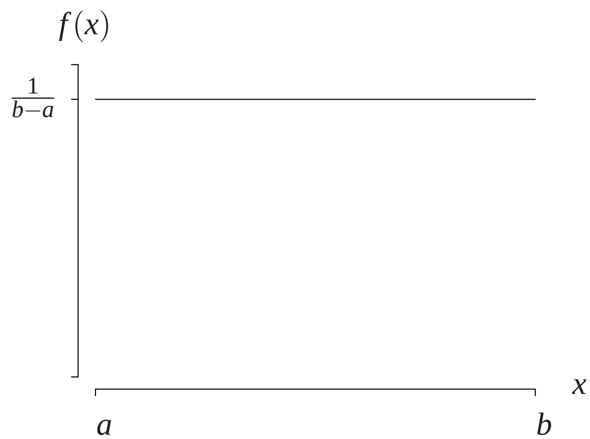
Uniform distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim U(a,b)$ is used to indicate that the random variable X has the uniform distribution with minimum a and maximum b. A uniform random variable X has probability density function

$$f(x) = \frac{1}{b-a} \qquad a < x < b,$$

for $-\infty < a < b < \infty$. The uniform distribution is used to model a random variable that is equally likely to occur between a and b. The uniform distribution is central to random variate generation. The uniform distribution would be an appropriate model for the position of the puncture on a flat tire. Event times associated with a Poisson process are uniformly distributed over an interval. The probability density function is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \frac{x-a}{b-a} \qquad a < x < b.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \frac{b-x}{b-a} \qquad a < x < b.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{1}{b-x} \qquad a < x < b.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = \ln(b-a) - \ln(b-x) \qquad a < x < b.$$

The inverse distribution function of X is

$$F^{-1}(u) = a + u(b-a) \qquad 0 < u < 1.$$

The median of X is

$$\frac{1}{2}(a+b).$$

The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = \begin{cases} 1 & t = 0 \\ \frac{e^{bt}-e^{at}}{t(b-a)} & t \neq 0. \end{cases}$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itX}\right] = \begin{cases} 1 & t = 0 \\ \frac{e^{ibt}-e^{iat}}{it(b-a)} & t \neq 0. \end{cases}$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \frac{1}{2}(a+b) \qquad V[X] = \frac{1}{12}(b-a)^2 \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = 0 \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{9}{5}.$$

APPL verification: The APPL statements

```
X := UniformRV(a, b);
CDF(X);
SF(X);
HF(X);
CHF(X);
IDF(X);
MGF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the cumulative distribution, survivor function, hazard function, cumulative hazard function, inverse, moment generating function, population mean, variance, skewness, kurtosis.