Theorem Random variates from the triangular distribution with minimum a, mode c, and maximum b can be generated in closed-form by inversion.

Proof The triangular(a, c, b) distribution has probability density function

$$f(x) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & a < x < c \\ \frac{2(b-x)}{(b-a)(b-c)} & c \le x < b \end{cases}$$

and cumulative distribution function

$$F(x) = \begin{cases} \frac{(x-a)^2}{(b-a)(c-a)} & a < x < c \\ 1 - \frac{(x-b)^2}{(b-a)(b-c)} & c \le x < b. \end{cases}$$

Equating the cumulative distribution function to u, where $0 < u < 1$ yields an inverse cumulative distribution function

$$F^{-1}(u) = \begin{cases} a + \sqrt{(b-a)(c-a)u} & 0 < u < \frac{c-a}{b-a} \\ b - \sqrt{(b-a)(b-c)(1-u)} & \frac{c-a}{b-a} \le u < 1. \end{cases}$$

So a closed-form variate generation algorithm using inversion for the triangular(a, c, b) distribution is

generate $U \sim U(0, 1)$
if $(U < (c - a)/(b - a))$ then
  $X \leftarrow a + \sqrt{(b-a)(c-a)U}$
else
  $X \leftarrow b - \sqrt{(b-a)(b-c)(1-U)}$
endif
return(X)

APPL illustration: The APPL statements

```
X := TriangularRV(0, 2, 5);
CDF(X);
IDF(X);
```

produce the inverse distribution function of a triangular random variable.